A Hyper-Heuristic Using GRASP with Path-Relinking: A Case Study of the Nurse Rostering Problem

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ABSTRACT

The goal of hyper-heuristics is to design and choose heuristics to solve complex problems. The primary motivation behind the hyper-heuristics is to generalize the solving ability of the heuristics. In this paper, the authors propose a Hyper-heuristic using GRASP with Path-Relinking (HyGraspPr). HyGraspPr generates heuristic sequences to produce solutions within an iterative procedure. The procedure of HyGraspPr consists of three phases, namely the construction phase, the local search phase, and the path-relinking phase. To show the performance of the HyGraspPr, the authors use the nurse rostering problem as a case study. The authors use an existing simulated annealing based hyper-heuristic as a baseline. The experimental results indicate that HyGraspPr can achieve better solutions than SAHH within the same running time and the path-relinking phase is effective for the framework of HyGraspPr.

Keywords: GRASP, Heuristics, Hyper-Heuristics, Nurse Rostering Problem, Path-relinking

INTRODUCTION

Hyper-heuristics aim to design general solving technologies for various problems by choosing existing heuristics (Burke, Hyde, Kendall, Ochoa, Ozcan, & Woodward, 2010). In contrast to meta-heuristics focused on the domain knowledge, hyper-heuristics tend to produce the High Level Heuristics (HLHs) for guiding the Low Level Heuristics (LLHs) (Burke, Hyde, Kendall, Ochoa, Ozcan, & Qu, 2010). The high level heuristics are referred to the heuristics designed by algorithm experts over the problem domains while the LLHs are referred to the heuristics designed by the problem domain experts. Since the domain knowledge is necessary for a particular problem and is hard to explore by an algorithm designer (Ochoa, Qu, & Burke, 2009), the primary motivation behind the hyper-heuristics is to help the algorithm designers to jump out of the limit from the problem domain and to produce general

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approaches. Based on the ability of general problem solving, hyper-heuristics have been applied to many kinds of problems, especially NP-hard problems, such as the timetabling (Burke, McCollum, Meisels, Petrovic, & Qu, 2007; Qu & Burke, 2009), the cutting stock (Terashima-Martin, Moran-Saavedre, & Ross, 2005), the workforce scheduling (Remde, Cowling, Dahal, & Colledge, 2006; Remde, Dahal, Cowling, & Colledge, 2009) and the p-median (Ren, Jiang, Xuan, & Luo, 2010).

In general, the goal of a hyper-heuristic is to design HLH to find an optimal LLH sequence, which can generate optimal solutions to the problems. As one kind of heuristics, most of hyper-heuristics draw on the experiments from the existing meta-heuristics, e.g., a simulated annealing based hyper-heuristic (Dowsland, Soubeiga, & Burke, 2007) and a genetic algorithm based hyper-heuristic (Ross, Martin-Blazquez, Schulenburg, & Hart, 2003). However, the kinds of hyper-heuristics are much fewer than those of meta-heuristics. The insufficiency of hyper-heuristics has limited the development of hyper-heuristics (Burke, Hyde, Kendall, Ochoa, Ozcan, & Woodward, 2010).

Greedy Randomized Adaptive Search Procedure (GRASP) with path-relinking is one of the effective meta-heuristics for problem solving (Resende & Ribeiro, 2003). There is no hyper-heuristic based on GRASP with path-relinking. As a typical meta-heuristic, GRASP with path-relinking is an iterative procedure to find the optimal solution. GRASP with path-relinking consists of three phases, such as the construction phase, the local search phase, and the path-relinking phase.

Motivated by the success of this algorithm in meta-heuristics, we propose a Hyper-heuristic using GRASP with Path-Relinking (HyGrasPr) in this paper. Our algorithm, HyGrasPr, generates LLH sequences to produce solutions in an iterative procedure. In each iteration, HyGrasPr builds an initial LLH sequence and applies a local search operator to find a relatively good LLH sequence. To avoid to be trapped as a local optimal LLH sequence, the path-relinking strategy is applied to obtain potential good solutions. To show the experimental results of HyGrasPr, we take the nurse rostering problem as a case study. On this problem, an existing simulated annealing based hyper-heuristic (SAHH) is employed as an experiment baseline. The results indicate that HyGrasPr can achieve better solutions than SAHH within the same running time.

The rest of this paper is organized as follows. First, we give the background of our work. We then propose the details of our HyGrasPr and present the experiments results with a case study on the nurse rostering problem.

BACKGROUND

Hyper-Heuristics and Meta-Heuristics

Hyper-heuristic technology is able to handle a wide range of problem domains rather than current meta-heuristic technology concentrated on a particular problem or a narrow class of problems (Burke, Kendall, Newall, Hart, Ross, & Schulenburg, 2003). A solution of a meta-heuristic is a structure abstracted from the problem domain; on the other hand, a solution of a hyper-heuristic is a sequence of LLHs. Many hyper-heuristics are based on the mechanism from meta-heuristics, e.g., hyper-heuristics based on tabu search for timetabling and rostering (Burke, Kendall, & Soubeiga, 2003), simulated annealing for determining shipper sizes (Dowsland et al., 2007), for automated planograms (Bai & Kendall, 2005), genetic algorithm (bin-packing) (Ross et al., 2003), and for 2D-regular cutting stock problems (Terashima-Martin et al., 2005), genetic programming for two dimensional strip packing (Burke, Hyde, Kendall, Ochoa, Ozcan, & Qu, 2010), and ant colony optimization for project presentation scheduling (Burke, Kendall, Silva, O’Brien, & Soubeiga, 2005) and for p-median by Ren et al. (2010). Moreover, Bai, Burke and Kendall (2008) investigate both the meta-heuristics and hyper-heuristics for fresh produce inventory control and shelf space allocation. Considering the similarity
between meta-heuristics and hyper-heuristics, many experiments from meta-heuristics can be employed to guide the hyper-heuristics design. Thus in this paper, we introduce another meta-heuristic to serve for hyper-heuristics, i.e., GRASP with path-relinking.

**GRASP with Path-Relinking**

GRASP is a multi-start meta-heuristic, in which each iteration consists a construction phase and a local search phase while path-relinking is a strategy to provide various solutions by “relinking” existing solutions (Resende & Ribeiro, 2003). For some of the applications, path-relinking is combined with GRASP to conduct a post-optimization or an intensification strategy to each local optimum (Festa, Pardalos, Pitsoulis, & Resende, 2006). Path-relinking is an enhancement strategy to build paths between solutions and to find other relevant solutions in the paths. Since time-consuming in practice, path-relinking is not applied at each GRASP iteration, but only periodically (Resende & Ribeiro, 2003).

**HYPER-HEURISTIC USING GRASP WITH PATH-RELINKING**

**Framework**

In this section, we present the details of our HyGrasPr. The solution generated by HyGrasPr is an LLH sequence, which can be applied to the problem instance in order to obtain the final solution to the problem. The LLH sequence is generated by an iterative procedure of GRASP with the path-relinking phase. In HyGrasPr, we first build an LLH sequence with the fixed length in GRASP construction phase. Then, we search the neighborhood of the LLH sequence for obtaining the local optimal LLH sequence (Croes, 1958). If an LLH sequence gets the best solution, we store it as the current best LLH sequence. Next, we use the current best LLH sequence and the local optimal LLH sequence to conduct the path-relinking phase. In each iteration, we apply the local optimal LLH sequence to the current solution to find the optimal LLH sequence for the problem. In our framework, we give three parameters, namely the number of iteration, the length of LLH sequence in each iteration, and the controllable parameter for the greedy or random strategies.

The pseudo-code for HyGrasPr is presented in Table 1. The kernel idea of our algorithm is to find the optimal LLH sequence by combining GRASP with path-relinking. In our implementation, we store the current best LLH sequence, which can be used for guiding problem solving in the path-relinking phase. Specifically, after a particular number of GRASP iterations, the local optimal LLH sequence is compared with the current best LLH sequence found by now. If the two LLH sequences are different with each other, we call the path-relinking phase. At the end of each iteration, the current best LLH sequence is updated for the next iteration. Existing work on the path-relinking for the meta-heuristics shows that the input length for the path-relinking can increase the running time (Resende & Ribeiro, 2003). In this work, the length of LLH sequence is determined according to an empirical result.

Each iteration of our HyGrasPr begins with an initial solution \( S_{\text{begin}} \) as an input and ends with a new solution \( S_{\text{next}} \) as an output. The new solution \( S_{\text{next}} \) is obtained by applying an LLH sequence to the initial solution \( S_{\text{begin}} \). The goal of the hyper-heuristics is to find an optimal LLH sequence, which is applied to the original problem. We denote the LLH sequence as \( Seq \). The final sequence of HyGrasPr is combined by all the LLH sequence obtained for each iteration.

**GRASP**

Based on an LLH sequence with the length \( N \), the hyper-heuristic may provide various solutions for the original problem. To generate an optimal LLH sequence, the GRASP procedure works with two parameters, namely used to denote the length of the LLH sequence and to conduct the construction of the restricted candidate list.
In general, each iteration of GRASP consists of two phases, namely the construction phase and the local search phase. We describe the construction phase and the local search phase for GRASP in our HyGrasPr. The construction phase of HyGrasPr is different from the typical GRASP construction for the metaheuristics. The output of the construction phase is an LLH sequence, which is build from an empty sequence. In each iteration of the construction phase, we add an LLH to the current LLH sequence until the LLH sequence length reaches the particular value $N$. The LLH to be added is randomly selected from the Restricted Candidate List (RCL). The RCL is a subset of the set of all candidate LLHs. If an LLH can generate a solution according with the given threshold, the LLH will be added to the RCL. The threshold is defined as $c_{\text{min}} + \alpha(c_{\text{max}} - c_{\text{min}})$, where $c_{\text{min}}$ and $c_{\text{max}}$ are the minimum and maximum of the evaluation function values for a new solution after applying the LLH to the current solution. The parameter $\alpha$ is a controllable parameter and $0 \leq \alpha \leq 1$. If $\alpha = 0$, the construction phase can be viewed as a greedy construction while if $\alpha = 1$, the construction phase can be viewed as a random construction. We present the pseudo-code of the construction phase in Table 2.

<table>
<thead>
<tr>
<th>Table 1. Pseudo-code for HyGrasPr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Procedure GRASP with path-relinking</td>
</tr>
<tr>
<td>Input: maximum iteration $\text{Iteration}$,</td>
</tr>
<tr>
<td>$Seq$ length $N$,</td>
</tr>
<tr>
<td>parameter used in construction $\alpha$,</td>
</tr>
<tr>
<td>call path-relinking frequency $PF$</td>
</tr>
<tr>
<td>Output: Solution $S$</td>
</tr>
<tr>
<td>1 for $i = 1, \ldots, \text{Iteration}$ do</td>
</tr>
<tr>
<td>2 $Seq = \text{GRASP}_\text{Construction}(N, \alpha, S_{\text{begin}}, S_{\text{next}})$</td>
</tr>
<tr>
<td>3 $Seq_{\text{local}} = \text{Local}_\text{Search}(N, S_{\text{begin}}, S_{\text{next}}, Seq)$</td>
</tr>
<tr>
<td>4 if $i % PF = 0$ then</td>
</tr>
<tr>
<td>5 Path-Relinking($Seq_{\text{best}}, Seq_{\text{local}}, S_{\text{begin}}, S_{\text{next}}$)</td>
</tr>
<tr>
<td>6 endif</td>
</tr>
<tr>
<td>7 update $Seq_{\text{best}}$</td>
</tr>
<tr>
<td>8 $S_{\text{begin}} \leftarrow S_{\text{nextNew}}$</td>
</tr>
<tr>
<td>9 endfor</td>
</tr>
</tbody>
</table>

After the construction phase of the HyGrasPr, we apply a local search phase to optimize the current LLH sequence. The 2-opt search is employed in the local search phase. To find the optimal LLH sequence, each LLH in the $Seq$ is traversed to search for the opportunity to build the final optimal LLH sequence. In each step, we change the selected LLH with other candidate LLHs, and then apply the new LLH sequence $Seq_{\text{temp}}$ to the solution $S_{\text{begin}}$. If the new $Seq_{\text{temp}}$ in the neighborhood could obtain a better solution, we replace $Seq_{\text{local}}$ with $Seq_{\text{temp}}$. After applying the current best LLH sequence to the solution $S_{\text{begin}}$, the new obtained solution is used as the initial solution in the next iteration. In Table 3, we present the pseudo-code of the local search.

**Path-Relinking**

In this section, we propose the path-relinking phase for HyGrasPr. Path-relinking can be viewed as an enhancement to the GRASP procedure. The main idea of the path-relinking is to explore the trajectories that connect an initial LLH sequence to a guiding LLH sequence. The initial LLH sequence is the input of the path-relinking while the guiding LLH sequence is the objective to guide the exploration. In HyGrasPr, a move of the path-relinking phase is
defined as a change from one LLH to another for reducing the difference between the guiding LLH sequence and the initial LLH sequence. In our experiments, the initial LLH sequence is the best LLH sequence found up till now and the guiding LLH sequence is the local optimal LLH sequence in current iteration. The initial LLH sequence can be defined as $Seq_i$ and the guiding solution can be defined as $Seq_g$. We present the pseudo-code for this phase in Table 4.

### A CASE STUDY OF THE NURSE ROSTERING PROBLEM

#### The Nurse Rostering Problem

The nurse rostering problem is a typical problem in the family of the personnel scheduling problem (Burke, Causmaecker, Berghe, & Landeghem, 2004). The goal of the nurse rostering problem is to decide the assignment of each nurse working over a specific planning period subject to various constraints (Burke, Li, & Qu, 2010). In this paper, we take the nurse rostering problem as a case study to investigate the performance of our HyGrasPr. The benchmark instances can be obtained from the nurse rostering problem website (http://www.cs.nott.ac.uk/~tec/NRP/). These instances in our experiments are generated from the real-world scenarios. All the constraints in the nurse rostering problem instances are modeled as the weighted objectives (Curtois, Ochoa, Hyde, & Vazquez-Rodriguez, 2010). The mathematical formulation of these objectives is described in (Burke, Curtois, Qu, & Vanden-Berghe, 2008).

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Table 2. Pseudo-code for construction of GRASP in HyGrasPr

<table>
<thead>
<tr>
<th>Procedure construction</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input:</strong> LLH sequence length $N$, parameter $\alpha$, the initial problem solution $Seq_{\text{init}}$ &lt;br&gt;<strong>Output:</strong> LLH sequence $Seq$, a new problem solution $Seq_{\text{next}}$</td>
</tr>
</tbody>
</table>

| 1 $Seq \leftarrow \Phi$, 2 best value by LLH sequence of current Iteration $Best \leftarrow \infty$ 3 while the length of $Seq < N$ 4 for every LLH $H_i \in H$ (the set of low level heuristics) do 5 apply $H_i$ to current solution $S_{\text{current}}$ get new solution $S_i$ 6 $C_i = \text{objective value of } S_i$ 7 endfor 8 $C^{\text{max}} \leftarrow \max \{ c_i | c_i \in C \}$ 9 $C^{\text{min}} \leftarrow \min \{ c_i | c_i \in C \}$ 10 the restricted candidate list $RCL \leftarrow \{ i \in H | c_i \leq C^{\text{min}} + \alpha(C^{\text{max}} - C^{\text{min}}) \}$ 11 randomly select a heuristic num $h$ from $RCL$ 12 if $c_h < Best$ then 13 $Best = c_h$ 14 endif 15 add $h$ to $Seq$ 16 $S_{\text{current}} \leftarrow S_h$ 17 endwhile 18 $Seq_{\text{next}} \leftarrow Seq_{\text{current}}$ |
Table 3. Pseudo-code of local search of GRASP in HyGrasPr

<table>
<thead>
<tr>
<th>Procedure local search</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input: LLH sequence length $N$, LLH sequence $Seq$, initial problem solution $S_{\text{begin}}$*, best value by LLH sequence of current Iteration $Best$, $S_{\text{next}}$, got from construction phase</td>
</tr>
<tr>
<td>Output: local optimal LLH sequence $Seq_{\text{local}}$, problem solution $S_{\text{next}}$</td>
</tr>
</tbody>
</table>

1 $Seq_{\text{local}} \leftarrow Seq$
2 for $i = 0,...,N$ do
3 for $j = 0,...,H_{\text{num}}$ (the number of low level heuristics) do
4 if the $i$th LLH of $Seq = j$ then
5 get a new LLH sequence $Seq_{\text{temp}}$ by replacing $i$th LLH of $Seq$ with $j$
6 $S_{\text{temp}} \leftarrow S_{\text{begin}}$
7 $localOptimalFlag = \text{false}$
8 for $k = 0,...,N$ do
9 apply $k$th LLH of $S_{\text{temp}}$ to $S_{\text{temp}}$
10 compute the objective value of $S_{\text{temp}}$
11 if value $< Best$ then
12 $Best = value$
13 $Seq_{\text{local}} \leftarrow S_{\text{temp}}$
14 $localOptimalFlag = \text{true}$
15 endfor
16 if $localOptimalFlag$ then
17 $S_{\text{next}} \leftarrow S_{\text{temp}}$
18 endif
19 endif
20 endfor
21 endfor

Table 4. Pseudo-code of path-relinking in HyGrasPr

<table>
<thead>
<tr>
<th>Procedure path-relinking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input: initial solution $Seq$, guiding solution $Seq_g$</td>
</tr>
<tr>
<td>Output: the best LLH sequence $Seq_{\text{best}}$</td>
</tr>
</tbody>
</table>

1 compute the sum Number of different LLHs between $Seq$ and $Seq_g$
2 save the different LLHs in the move set, $M$
3 for $i = 0,...,\text{Number}$ do
4 find the bestmove in the move set, $M$
5 apply the bestmove
6 update the $Seq_{\text{best}}$
7 delete the selected bestmove from $M$.
8 endfor
To show the performance of our HyGrasPr, we use 12 LLHs in this paper. These LLHs are obtained in the hyper-heuristic competition website (http://www.asap.cs.nott.ac.uk/chesc2011/index.html). A detailed description of these heuristic is reproduced here for completeness (Curtois et al., 2010).

- $h_1$, local search heuristic using “hill climbers” with vertical neighborhood operator (moving shifts vertically between two employees in the roster);
- $h_2$, local search heuristic using “hill climbers” with horizontal neighborhood operator (moving shifts horizontally in single employee’s work pattern in the roster);
- $h_3$, local search heuristic using “hill climbers” with new neighborhood operator (introducing new shifts or deleting shifts into the roster);
- $h_4$, local search heuristic, variable depth search using new moves as links in the ejection chain;
- $h_5$, local search heuristic, variable depth search using new moves as links in the ejection chain and test replacing an entire work pattern for a single employee as a link in the chain;
- $h_6$, ruin and recreate heuristic, un-assigning $x = \text{Round}\left(\beta \times 4\right) + 2$ schedules ($\beta$ is the intensity of mutation and $\text{Round}()$ is the rounding function). The heuristic first randomly selects $x$ employee’s schedules and un-assigning all the shifts in them. They are rebuilt by first satisfying objectives related to requests to work certain days or shifts and then by satisfying objectives related to weekends;
- $h_7$, same as $h_6$ but “$x = \text{Round}\left(\beta \times \text{Number of employees in roster}\right)$”;
- $h_8$, same as $h_6$ but creating a small perturbation in the solution by using $x = 1$;
- $h_9$, crossover heuristic, choosing the best $x$ assignments in each parent and make these assignments in the offspring;
- $h_{10}$, crossover heuristic, it creates a new roster by using all the assignments made in the parents. It makes those that are common to both parents first and then alternately selects an assignment from each parent and makes it in the offspring unless the cover objective is already satisfied;
- $h_{11}$, crossover heuristic, creating the new roster by making assignments which are only common to both parents;
- $h_{12}$, mutation heuristic, randomly mutating a selected roster.

**Results**

We show the performance of our HyGrasPr in the experiments. All the experiments are run on a PC with Intel Core Duo 2.53 GHz and 4GB RAM running on a Microsoft Windows 7 Ultimate.

Before showing the experimental results, we give some experiments on the parameter values and performance. We test our algorithm on 4 typical problem instances (ORTEC02, Ikegami-3Shift-DATA1, Ikegami-3Shift-DATA1.1, and Ikegami-3Shift-DATA1.2) with different parameters. The parameter $\alpha$ which is the probability of choosing RCL in GRASP is set to 0.3. The parameter $N$, the length of LLH sequence is set to 5, 10, 15, and 20, respectively. We limit the time bound to 30 minutes. The results are shown in Table 5.

It is showed that when the parameter $\alpha$ is fixed to 0.3, the LLH sequence length with the value 15 gets the best result. In general, for the length from 5 to 15, the objective function of solutions is decreasing when the length increases. That is the length with the value 15 may bring better solutions than others. On the other hand, when the sequence length increases, the running time may increase. Therefore, the experiment with the length of the value 20 can cost more time than the previous three experiments. Since we limit the time bound, the experiment with the length 20 cannot finish the process of searching. Thus, we choose 15 as the sequence length in the following experiments.

We study the parameter $\alpha$ with the values among 0.1, 0.3, 0.5, and 0.7. We also limit the
time bound to 30 minutes. The results are showed in Table 6.

In the structure of the construction phase, the parameter $\alpha$ is a trade-off between greedy and random. From the results, we can see that the value 0.3 maybe a good value for the instances in our experiments.

The experiments of adjusting the parameters in our HyGrasPr on the nurse rostering problem above show that $\alpha = 0.3$ and $N = 15$ can make our HyGrasPr perform well. Then we compare our hyper heuristic with an existing algorithm on the instances.

### Table 5. Objective functions for HyGrasPr with different sequence length on 4 typical instances

<table>
<thead>
<tr>
<th>Sequence length</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>ORTEC02</td>
<td>335.0</td>
<td>350.0</td>
<td>310.0</td>
<td>335.0</td>
</tr>
<tr>
<td>Ikegami-3Shift-DATA1</td>
<td>18.0</td>
<td>11.0</td>
<td>17.0</td>
<td>22.0</td>
</tr>
<tr>
<td>Ikegami-3Shift-DATA1.1</td>
<td>23.0</td>
<td>21.0</td>
<td>13.0</td>
<td>21.0</td>
</tr>
<tr>
<td>Ikegami-3Shift-DATA1.2</td>
<td>20.0</td>
<td>23.0</td>
<td>18.0</td>
<td>20.0</td>
</tr>
</tbody>
</table>

### Table 6. Objective functions for HyGrasPr with different $\alpha$ on 4 typical instances

<table>
<thead>
<tr>
<th>Value of $\alpha$</th>
<th>0.1</th>
<th>0.3</th>
<th>0.5</th>
<th>0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>ORTEC02</td>
<td>365.0</td>
<td>310.0</td>
<td>330.0</td>
<td>380.0</td>
</tr>
<tr>
<td>Ikegami-3Shift-DATA1</td>
<td>22.0</td>
<td>17.0</td>
<td>13.0</td>
<td>15.0</td>
</tr>
<tr>
<td>Ikegami-3Shift-DATA1.1</td>
<td>20.0</td>
<td>13.0</td>
<td>15.0</td>
<td>31.0</td>
</tr>
<tr>
<td>Ikegami-3Shift-DATA1.2</td>
<td>19.0</td>
<td>18.0</td>
<td>21.0</td>
<td>23.0</td>
</tr>
</tbody>
</table>

### Table 7. Results for SAHH, HyGrasPr, and HyGrasPr without path-relinking

<table>
<thead>
<tr>
<th>Problem instance</th>
<th>Time(s)</th>
<th>SAHH</th>
<th>HyGrasPr</th>
<th>HyGrasPr without path-relinking</th>
</tr>
</thead>
<tbody>
<tr>
<td>BCV-3.46.1</td>
<td>17870</td>
<td>3321.0</td>
<td>3307.0</td>
<td>3322.0</td>
</tr>
<tr>
<td>BCV-A.12.2</td>
<td>12012</td>
<td>2210.0</td>
<td>2005.0</td>
<td>2340.0</td>
</tr>
<tr>
<td>ORTEC02</td>
<td>4972</td>
<td>395</td>
<td>300.0</td>
<td>440.0</td>
</tr>
<tr>
<td>Ikegami-3Shift-DATA1</td>
<td>5981</td>
<td>12.0</td>
<td>11.0</td>
<td>21.0</td>
</tr>
<tr>
<td>Ikegami-3Shift-DATA1.1</td>
<td>6315</td>
<td>15.0</td>
<td>13.0</td>
<td>25.0</td>
</tr>
<tr>
<td>Ikegami-3Shift-DATA1.2</td>
<td>5990</td>
<td>15.0</td>
<td>14.0</td>
<td>36.0</td>
</tr>
<tr>
<td>CHILD-A2</td>
<td>43331</td>
<td>1103.0</td>
<td>1108.0</td>
<td>1240.0</td>
</tr>
<tr>
<td>ERRVH-A</td>
<td>20022</td>
<td>2165.0</td>
<td>2156.0</td>
<td>2344.0</td>
</tr>
<tr>
<td>ERRVH-B</td>
<td>16948</td>
<td>3167.0</td>
<td>3167.0</td>
<td>4093.0</td>
</tr>
<tr>
<td>MER-A</td>
<td>38369</td>
<td>9289.0</td>
<td>8934.0</td>
<td>14964.0</td>
</tr>
</tbody>
</table>
We choose 10 typical problem instances from the nurse rostering problem instances. We make comparison with the Simulated Annealing Hyper-Heuristic (SAHH) (Bai, Blazewicz, Burke, Kendall, & McCollum, 2007) proposed by Bai et al. SAHH is a hyper heuristic which includes a stochastic heuristic selection mechanism, a simulated annealing acceptance criterion, and a short-term memory. SAHH learns priorities of different low level heuristics from their historical performance and starts to select them depending on priorities. Therefore, the heuristics with good performance have higher priorities and are more likely to be chosen. The simulated annealing acceptance criterion is used to determine whether to be applied the selected heuristic. Information about the acceptance decisions by the acceptance criterion is then fed back to the heuristic selection mechanism in order to make good better decisions in the future. In (Bai et al., 2007), it was shown that SAHH is a well performed hyper-heuristic. SAHH can also be used for the nurse rostering problem. The number of iterations of SAHH is set to $K = 12000$ as described in (Bai et al., 2007). On each instance, the running time of our HyGrasPr is limited to the used time of SAHH. In Table 7, we present the results for HyGrasPr, SAHH and HyGrasPr without Path-relinking.

From the result we can conclude that our hyper heuristic can outperform the existing algorithm, SAHH. Among the 10 instances, HyGrasPr can get 8 better solutions out of 10 instances. We also compare HyGrasPr with the hyper heuristic implemented by GRASP without path-relinking. The performance of HyGrasPr is much better than that of HyGrasPr without path-relinking. This result shows that the path-relinking mechanism is very useful in our framework. In the path-relinking phase, the process the guidance of the LLH sequence from the initial one to the guiding one can provide various LLH sequences as the candidate LLH sequences. The performance of HyGrasPr is also better than that of SAHH.

To show the details of our experiments, we analyze the solving procedure of two algorithms, SAHH and HyGrasPr. Figure 1 presents the details of the processes when solving the instance ORTEC02 using SAHH and HyGrasPr. The results show that our algorithm HyGrasPr can improve solution step by step while SAHH provides a jumping curve.

**Figure 1. Solving procedures of SAHH and HyGrasPr on the ORTEC02 instance**
At the same time for both the algorithms, the solutions in SAHH are not as better as those in HyGrasPr. Our HyGrasPr provides a much stronger searching ability.

CONCLUSION AND FUTURE WORK

In this paper, we propose a Hyper-heuristic using GRASP with Path-Relinking (HyGrasPr). The procedure of HyGrasPr consists of three phases, such as the construction phase, the local search phase, and the path-relinking phase. To present the experimental results, we give a case study on the nurse rostering problem. The results show that HyGrasPr can achieve better solutions on 8 out of 10 instances than the existing simulate annealing hyper-heuristic.

In future work, we plan to give an empirical study on the performance of our HyGrasPr. Since the mechanism behind HyGrasPr is a little time-consuming, we want to design some improvement to reduce the running time. Another future work is to apply our HyGrasPr to some other problems. For example, we plan to give a comparison among the family of the nurse rostering problem. This work can show the generalization ability of our work.

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