

Graphs in Machine Learning

Michal Valko

Inria Lille - Nord Europe, France

TA: Pierre Perrault

Partially based on material by: Andreas Krause, Branislav Kveton, Michael Kearns

October 2nd, 2017

MVA 2017/2018

Piazza for Q&A's



Purpose

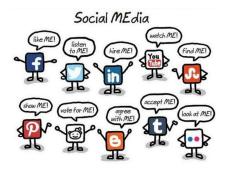
- registration for the class
- register with your school email and full name
- online course discussions and announcements
- questions and answers about the material and logistics
- students encouraged to answer each others' questions
- homework assignments
- virtual machine link and instructions
- draft of the slides before the class

https://piazza.com/ens_cachan/fall2017/mvagraphsml NO EMAILS!
class code given during the class

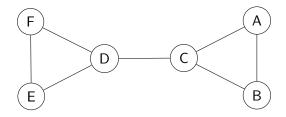


Graphs from social networks

- people and their interactions
- directed (Twitter) and undirected (Facebook)
- structure is rather a phenomena
- typical ML tasks
 - advertising
 - product placement
 - link prediction (PYMK)





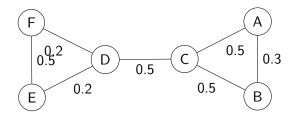




Maximizing the Spread of Influence through a Social Network http://www.cs.cornell.edu/home/kleinber/kdd03-inf.pdf

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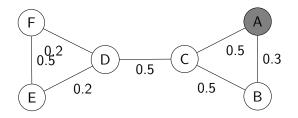
SequeL - 4/39



Who should get free cell phones?

 $V = \{A | ice, Bob, Charlie, Dorothy, Eric, Fiona\}$

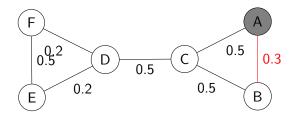




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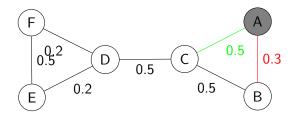




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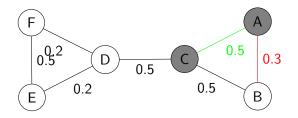




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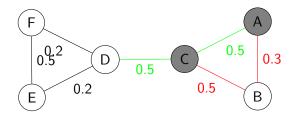




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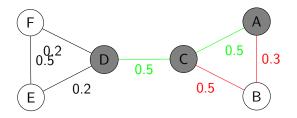




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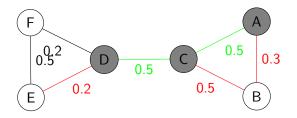




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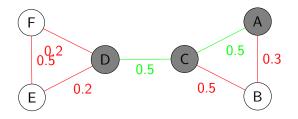




Who should get free cell phones?

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Who should get free cell phones?

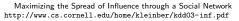
 $V = \{Alice, Bob, Charlie, Dorothy, Eric, Fiona\}$

F(S) = Expected number of people influenced when targeting

 $S\subseteq V$ under some propagation model - e.g., cascades

How would you choose the target customers?

highest degree, close to the center, . . .





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Submodularity: Definition

A set function on a discrete set A is submodular if for any $S \subseteq T \subseteq A$ and for any $e \in A \setminus T$

 $f(S \cup \{e\}) - f(S) \geq f(T \cup \{e\}) - f(T)$

Example: $S = {\text{stuff}} = {\text{bread, apple, tomato, ...}}$ f(V) = cost of getting products V

$$\begin{split} f(\{\text{bread}\}) &= c(\text{bakery}) + c(\text{bread}) \\ f(\{\text{bread}, \text{apple}\}) &= c(\text{bakery}) + c(\text{bread}) + c(\text{market}) + c(\text{apple}\}) \\ f(\{\text{bread}, \text{tomato}\}) &= c(\text{bakery}) + c(\text{bread}) + c(\text{market}) + c(\text{tomato}) \\ f(\{\text{bread}, \text{tomato}, \text{apple}\}) &= c(\text{bakery}) + c(\text{bread}) + c(\text{market}) + c(\text{tomato}) + c(\text{apple}) \end{split}$$

Adding an apple to the smaller set costs more!

 $bread \subseteq bread, tomato \}$

 $f(\{bread, apple\}) - f(\{bread\}) > f(\{bread, tomato, apple\}) - f(\{tomato, bread\})$

Diminishing returns: Buying in bulk is cheaper!

Submodularity: Application

Objective: Find $\arg \max_{S \subseteq A, |S| \le k} f(S)$

Property: NP-hard in general

Special case: f is also nonnegative and monotone.

Other examples: information, graph cuts, covering, ...

Link to our product placement problem on a social network graph?

submodular?, nonnegative?, monotone?, k?

http://thibaut.horel.org/submodularity/papers/nemhauser1978.pdf

Let $S^* = \arg \max_{S \subseteq A, |S| \le k} f(S)$ where f is monotonic and submodular set function and let S_{Greedy} be a greedy solution.

$$\mathsf{Then} \quad f(S_{\texttt{Greedy}}) \geq \left(1 - \tfrac{1}{e}\right) \cdot f(S^\star).$$



Submodularity: Greedy algorithm

1: Input:

- 2: k: the maximum allowed cardinality of the output
- 3: V: a ground set
- 4: f: a monotone, non-negative, and submodular function
- 5: Run:
- 6: $S_0 = \emptyset$
- 7: for i = 1 to k do
- 8: $S_i \leftarrow S_{i-1} \cup \left\{ \operatorname{arg\,max}_{a \in V \setminus S_{i-1}} \left[f\left(\{a\} \cup S_{i-1} \right) f\left(S_{i-1} \right) \right] \right\}$
- 9: end for
- 10: **Output:**
- 11: Return $S_{\text{Greedy}} = S_k$

Let $S^* = \arg \max_{S \subseteq A, |S| \le k} f(S)$ where f is monotonic and submodular set function and let S_{Greedy} be a greedy solution.

$$\mathsf{Then} \quad f(S_{\mathtt{Greedy}}) \geq ig(1 - rac{1}{e}ig) \cdot f(S^\star).$$



Submodularity: Approximation guarantee of Greedy Let S_i be the *i*-th set selected by Greedy, $S_{Greedy} = S_k$. We show

$$f(S^{\star}) - f(S_i) \leq \left(1 - \frac{1}{k}\right)^i \cdot f(S^{\star}).$$

Difference from the optimum before the *i*-th step

$$\begin{split} f\left(S^{\star}\right) - f\left(S_{i-1}\right) &\leq f\left(S^{\star} \cup S_{i-1}\right) - f\left(S_{i-1}\right) \\ &\leq \sum_{a \in S^{\star} \setminus S_{i-1}} \left(f\left(\{a\} \cup S_{i-1}\right) - f\left(S_{i-1}\right)\right) \\ &\leq \sum_{a \in S^{\star} \setminus S_{i-1}} \left(f\left(S_{i}\right) - f\left(S_{i-1}\right)\right) \\ &\leq k\left(f\left(S_{i}\right) - f\left(S_{i-1}\right)\right) \end{split}$$

Difference from the optimum after the *i*-th step

$$\begin{split} f(S^{\star}) - f(S_i) &= f(S^{\star}) - f(S_{i-1}) - (f(S_i) - f(S_{i-1})) \\ &\leq f(S^{\star}) - f(S_{i-1}) - \frac{f(S^{\star}) - f(S_{i-1})}{k} \end{split}$$

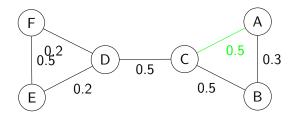
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Submodularity: Graph-related examples

- Influence maximization on networks (current example)
- Maximum-weight spanning trees
- Graph cuts
- Structure learning in graphical models (PGM course)
- More examples http://people.math.gatech.edu/~tetali/LINKS/IWATA/SFGT.pdf
- Deep Submodular Functions (2017) https://arxiv.org/pdf/1701.08939.pdf

back to the influence-maximization example



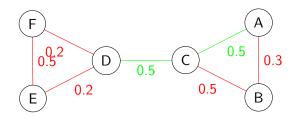


Key idea: Flip coins *c* in advance \rightarrow "live" edges

MIIA: http://hanj.cs.illinois.edu/pdf/dmkd12_cwang.pdf/ Tutorial: cf. Andreas Krause http://submodularity.org/ Course: Jeff Billmes at UW



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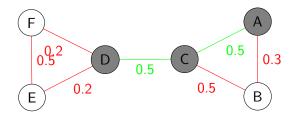


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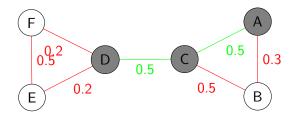
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Key idea: Flip coins *c* in advance \rightarrow "live" edges $F_c(V)$ = People influenced under outcome *c* (set cover!)

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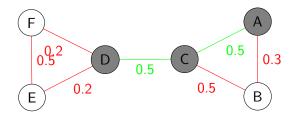




Key idea: Flip coins *c* in advance \rightarrow "live" edges $F_c(V) =$ People influenced under outcome *c* (set cover!) $F(V) = \sum_c P(c)F_c(V)$ is submodular as well!

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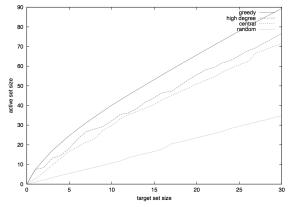
Computational issues?

MIIA: http://hanj.cs.illinois.edu/pdf/dmkd12_cwang.pdf/ Tutorial: cf. Andreas Krause http://submodularity.org/ Course: Jeff Billmes at UW



Success story #1 Product placement - comparison





greedy approximation does better than the centrality measures



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Graphs from utility and technology networks

- link services
- power grids, roads, transportation networks, Internet, sensor networks, water distribution networks
- structure is either hand designed or not
- typical ML tasks
 - best routing under unknown or variable costs
 - identify the node of interest



Berkeley's Floating Sensor Network



Graphs from information networks

web

blogs

- wikipedia
- typical ML tasks
 - find influential sources
 - search (PageRank)



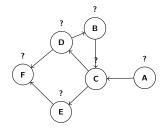
Blog cascades (ETH) - submodularity

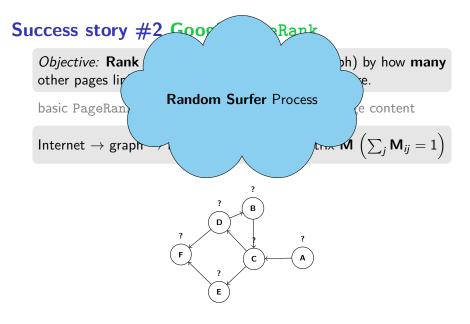


Objective: **Rank** all web pages (nodes on the graph) by how **many** other pages link to them and how **important** they are.

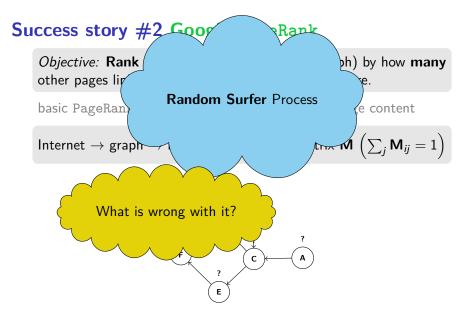
basic PageRank is independent of query and the page content

Internet \rightarrow graph \rightarrow matrix \rightarrow stochastic matrix $\mathsf{M}\left(\sum_{j}\mathsf{M}_{ij}=1\right)$









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http://infolab.stanford.edu/~backrub/google.html:

PageRank can be thought of as a model of user behavior. We assume there is a "random surfer" who is given a web page at random and keeps clicking on links, never hitting "back" but eventually gets bored and starts on another random page.

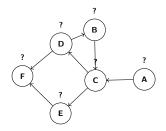
- page is important if important pages link to it
 - circular definition
- importance of a page is distributed evenly
- probability of being bored is 15%



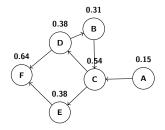
Google matrix: $\mathbf{G} = (1 - p)\mathbf{M} + p \cdot \frac{1}{N} \mathbb{1}_{N \times N}$, where p = 0.15



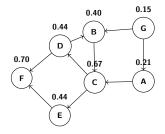
Google matrix: $\mathbf{G} = (1 - p)\mathbf{M} + p \cdot \frac{1}{N}\mathbb{1}_{N \times N}$, where p = 0.15 **G** is stochastic why? what is Ga for any a? We look for $\mathbf{Gv} = 1 \times \mathbf{v}$, steady-state vector, a right eigenvector with eigenvalue 1. why? **Perron's theorem:** Such v exists and it is unique if the entries of **G** are positive.



Google matrix: $\mathbf{G} = (1 - p)\mathbf{M} + p \cdot \frac{1}{N}\mathbb{1}_{N \times N}$, where p = 0.15 **G** is stochastic why? what is Ga for any a? We look for $\mathbf{Gv} = 1 \times \mathbf{v}$, steady-state vector, a right eigenvector with eigenvalue 1. why? **Perron's theorem:** Such v exists and it is unique if the entries of **G** are positive.



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History: [Desikan, 2006]

- The anatomy of a large-scale hypertextual web search engine [Brin & Page 1998]
- US patent for PageRank granted in 2001
- Google indexes 10's of billions of web pages (1 billion = 10^9)
- ▶ Google serves ≥ 200 million queries per day
- Each query processed by \geq 1000 machines
- All search engines combined process more than 500 million queries per day



Problem: Find an eigenvector of a stochastic matrix.

▶ $n = 10^9$!!!

- Iuckily: sparse (average outdegree: 7)
- better than a simple centrality measure (e.g., degree)
- power method

$$\mathbf{v}_0 = (\mathbf{1}_A \quad \mathbf{0}_B \quad \mathbf{0}_C \quad \mathbf{0}_D \quad \mathbf{0}_E \quad \mathbf{0}_F)^{\mathsf{T}}$$
$$\mathbf{v}_1 = \mathbf{G}\mathbf{v}_0$$
$$\mathbf{v}_{t+1} = \mathbf{G}\mathbf{v}_t = \mathbf{G}^{t+1}\mathbf{v}$$

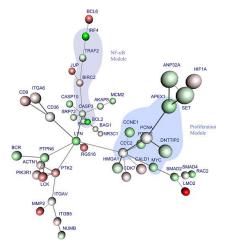
 $\mathbf{v}_{t+1} = \mathbf{v}_t \implies \mathbf{G}\mathbf{v}_t = \mathbf{v}_t$ and we found the steady vector

But wait, **M** is sparse, but **G** is dense! What to do?



Graphs from biological networks

- protein-protein interactions
- gene regulatory networks
- typical ML tasks
 - discover unexplored interactions
 - learn or reconstruct the structure



Diffuse large B-cell lymphomas - Dittrich et al. (2008)



graph is not naturally given



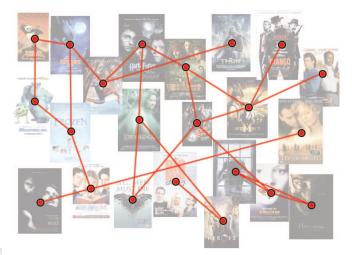


but we can construct it





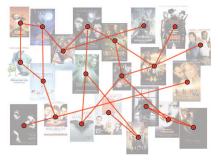
and use it as an abstraction





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- vision
- audio
- text
- typical ML tasks
 - semi-supervised learning
 - spectral clustering
 - manifold learning



movie similarity



Two sources of graphs in ML

Graph as models for networks

- given as an input
- discover interesting properties of the structure
- represent useful information (viral marketing)
- be the object of study (anomaly detection)

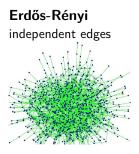
Graph as nonparametric basis

- we create (learn) the structure
- flat vectorial data \rightarrow similarity graph

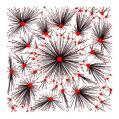
. . .

- nonparametric regularizer
- encode structural properties: smoothness, independence,

Random Graph Models



Barabási-Albert preferential attachment



Watts-Strogatz, Chung-Lu, Fiedler,

Stochastic Blocks

modeling communities





What will you learn in the Graphs in ML course?

Concepts, tools, and methods to work with graphs in ML.

Theoretical toolbox to analyze graph-based algorithms.

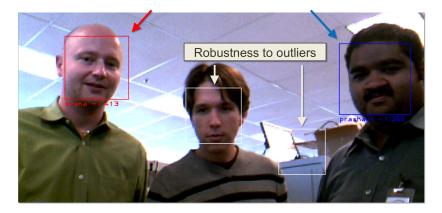
Specific applications of graphs in ML.

How to tackle: large graphs, online setting, graph construction

One example: Online Semi-Supervised Face Recognition

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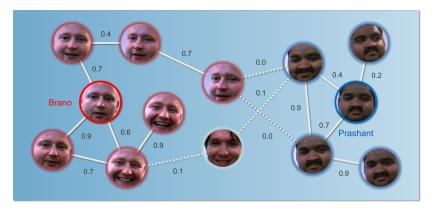
graph is not given



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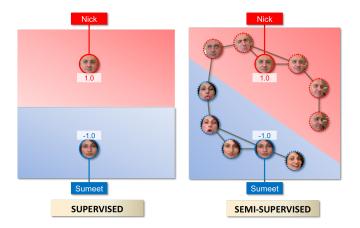
we will construct it!



An example of a similarity graph over faces. The faces are vertices of the graph. The edges of the graph connect similar faces. Labeled faces are outlined by thick solid lines.

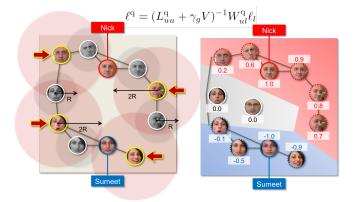


graph-based semi-supervised learning



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online learning - graph sparsification



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DEMO

second TD





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OSS FaceReco: Analysis

$$\frac{1}{n} \sum_{t} (\ell_{t}^{q}[t] - y_{t})^{2} \leq \frac{3}{n} \sum_{t} (\ell_{t}^{*} - y_{t})^{2} + \frac{3}{n} \sum_{t} (\ell_{t}^{o}[t] - \ell_{t}^{*})^{2} + \frac{3}{n} \sum_{t} (\ell_{t}^{q}[t] - \ell_{t}^{o}[t])^{2}$$
Error of our solution
Offline learning error
Quantization error
Quantization error

Claim: When the regularization parameter is set as $\gamma_g = \Omega(n_l^{3/2})$, the difference between the risks on labeled and all vertices decreases at the rate of $O(n_l^{-1/2})$ (with a high probability)

$$\frac{1}{n} \sum_{t} (\ell_{t}^{*} - y_{t})^{2} \leq \frac{1}{n_{t}} \sum_{i \in I} (\ell_{i}^{*} - y_{i})^{2} + \beta + \sqrt{\frac{2 \ln(2/\delta)}{n_{t}}} (n_{t}\beta + 4)$$
$$\beta \leq \left[\frac{\sqrt{2}}{\gamma_{g} + 1} + \sqrt{2n_{t}} \frac{1 - \sqrt{c_{u}}}{\sqrt{c_{u}}} \frac{\lambda_{M}(L) + \gamma_{g}}{\gamma_{g}^{2} + 1} \right]$$



OSS FaceReco: Analysis

$$\frac{1}{n} \sum_{t} (\ell_{t}^{q}[t] - y_{t})^{2} \leq \frac{3}{n} \sum_{t} (\ell_{t}^{*} - y_{t})^{2} + \frac{3}{n} \sum_{t} (\ell_{t}^{o}[t] - \ell_{t}^{*})^{2} + \frac{3}{n} \sum_{t} (\ell_{t}^{q}[t] - \ell_{t}^{o}[t])^{2}$$
Error of our Offline learning error Online learning error Quantization error

Claim: When the regularization parameter is set as $\gamma_g = \Omega(n^{1/4})$, the average error between the offline and online HFS predictions decreases at the rate of $O(n^{-1/2})$

$$\begin{split} \frac{1}{n} \sum_{\tau} \left(\ell_{\tau}^{\circ}[t] - \ell_{\tau}^{*} \right)^{2} &\leq \frac{1}{n} \sum_{\tau} \left\| \ell^{\circ}[t] - \ell^{*} \right\|_{2}^{2} \leq \frac{4n_{t}}{(\gamma_{g} + 1)^{2}} \\ & \left\| \ell \right\|_{2} \leq \frac{\left\| y \right\|_{2}}{\lambda_{m}(C^{-1}K + I)} = \frac{\left\| y \right\|_{2}}{\lambda_{m}(K)\lambda_{M}^{-1}(C) + 1} \leq \frac{\sqrt{n_{t}}}{\gamma_{g} + 1} \end{split}$$



OSS FaceReco: Analysis

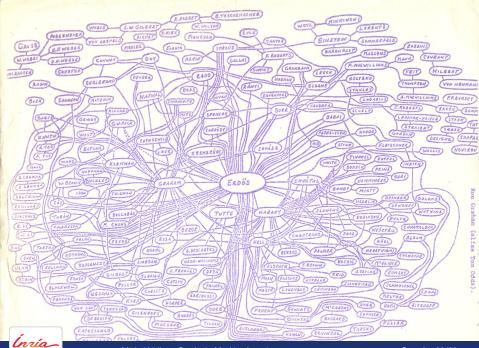
$$\frac{1}{n} \sum_{t} (\ell_{t}^{q}[t] - y_{t})^{2} \leq \frac{3}{n} \sum_{t} (\ell_{t}^{*} - y_{t})^{2} + \frac{3}{n} \sum_{t} (\ell_{t}^{o}[t] - \ell_{t}^{*})^{2} + \frac{3}{n} \sum_{t} (\ell_{t}^{q}[t] - \ell_{t}^{o}[t])^{2}$$
Error of our solution Offline learning error Online learning error Quantization error

Claim: When the regularization parameter is set as $\gamma_g = \Omega(n^{1/8})$, and the Laplacians L^q and L^o and normalized, the average error between the online and online quantized HFS predictions decreases at the rate of O(n^{-1/2})

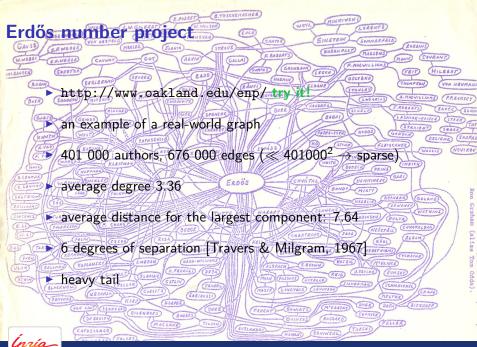
$$\frac{1}{n} \sum_{t} \left(\ell_{t}^{q}[t] - \ell_{t}^{o}[t] \right)^{2} \leq \frac{1}{n} \sum_{t} \left\| \ell^{q}[t] - \ell^{o}[t] \right\|_{2}^{2} \leq \frac{n_{t}}{c_{u}^{2} \gamma_{g}^{4}} \left\| L^{q} - L^{o} \right\|_{F}^{2}$$

 $\|L^q - L^e\|_F^2 \propto O(k^{-2/d})$ The distortion rate of online k-center clustering is $O(k^{-1/d})$, where d is dimension of the manifold and k is the number of representative vertices



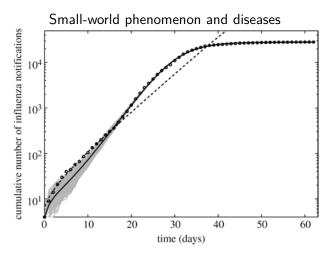


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Spanish flu in San Francisco 1918–1919



http://rsif.royalsocietypublishing.org/content/4/12/155

Small world: Obvious?

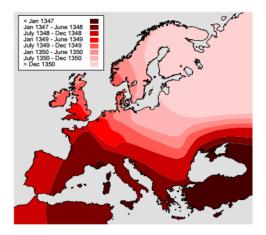
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Black death!



Ínría

Black death: spread



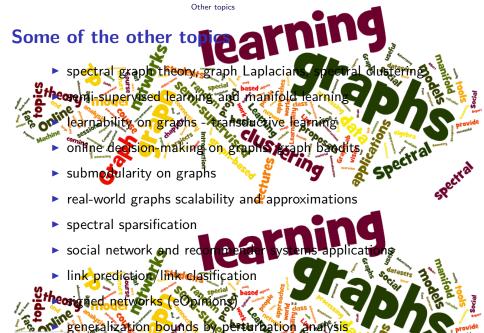
source: catholic.org

https://www.youtube.com/watch?v=EEK6c9Bh5CQ

Ínría_

Michal Valko - Graphs in Machine Learning

SequeL - 32/39



Michal Valko - Graphs in Machine Learning

Links to the other courses

Introduction to statistical learning

links to the learning theory on graphs: label propagation, learnability, generalization

Reinforcement learning

link to the online learning (bandit) lecture at the end of the semester

Advanced learning for text and graph data

- data-mining graph course on the topics not covered in this course
- details on the next slide



Administrivia

MVA and Graphs: 2 courses

The two MVA graph courses offer complementary material.

Fall: Graphs in ML

this class

- focus on learning
- spectral clustering
- random walks
- graph Laplacian
- semi-supervised learning
- manifold learning
- theoretical analyses
- online learning
- recommender systems

Xmas: ALTeGraD

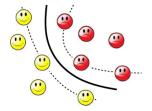
by Michalis Vazirgiannis

- dimensionality reduction
- feature selection
- text mining
- graph mining
- community mining
- graph generators
- graph-evaluation measures
- privacy in graph mining
- big data



Administrivia

Statistical Machine Learning in Paris!



https://sites.google.com/site/smileinparis/sessions-2016--17



DELTA: PhD proposal

Dynamically Evolving Long-Term Autonomy

- join project between 4 partners, UPF Barcelona, MUL Austria, ULG Belgium, and Inria
- Jonsson, Neu, Gomez, Valko, Kaufmann, Lazaric, Auer, Ortner, Cornelusse, Ernst
- PhD position at SequeL team at Inria
- project starts on 1.1.2018, PhD student expected to start September/October 2018
- 4 postdocs, one in each center
- Inria will lead the effort on adaptive planning with a model that can adapt to changes. Inria will work with MUL on the hierarchical state partitioning



Administrivia

- Time: Mondays 10h30-12h30
- Place: ENS Cachan Salle Condorcet
- **7-8 lectures:** 2.10. 9.10. 15.10. 30.10. 6.11. 20.11. 11.12. 18.12. **3 recitations (TDs):** 23.10. 13.11. 27.11. (14h-16h)
- **Validation:** grades from TDs (40%) + class project (60%) **Research:** contact me for *internships*, *PhD theses*, *projects*, etc.

Course website: http://researchers.lille.inria.fr/~valko/hp/mva-ml-graphs

Contact, online class discussions, and announcements: https://piazza.com/ens_cachan/fall2017/mvagraphsml class code given during the class



Administrivia

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