We are interested in optimizing a sample of a standard Brownian motion. More precisely, we want to sequentially select query points only polynomial rates. Our algorithm is adaptive—each query depends on previous results.

Theorem 1. With an approximation of its maximum using the smallest possible number of samples, one can achieve an accuracy of at least \(\varepsilon\).

Algorithm 1: OOB algorithm

1. Input: \(\varepsilon\)
2. Init: \(I \leftarrow \{0, 1\}, t_1 = W(1)\)
3. for \(i = 2, 3, 4, \ldots\) do
   4. \(\{a, b\} \in \arg \max_{t \in I} W_t\) (break if \(\varepsilon/2\) is reached)
   5. if \(\varepsilon/2 < \varepsilon\) then exit
   6. \(I \leftarrow I \setminus \{a, b\}\)
   7. end if
   8. \(t_i \leftarrow W\left(\frac{a + b}{2}\right)\)
   9. \(I \leftarrow I \cup \{a, b\}\)
10. end for

Output: location \(\tilde{t}_p, t_p\) where \(\max_{t \in I} W_t\) and its value \(\frac{W(\tilde{t}_p)}{\varepsilon}\)

Guarantees:

- Correctness: algorithm definition + the law of Brownian bridge
- Open problem: \(d\) and \(C(\varepsilon)\) t.l. - \(C(\varepsilon)\) s.t. Brownian \(\in (d, C(\varepsilon))\)
- Challenge: Brownian motion is a stochastic process!

Prior work:

- Al-Mharmah and Calvin (1996)
  - a non-adaptive method
  - sample complexity: \(1/\varepsilon\)
- Calvin (2017)
  - adaptive method
  - better than any polynomial
  - does not guarantee an exponential rate