We are interested in optimizing a sample of a standard Brownian motion on this is when we want to return a sample of the maximum (and the location of the maximum) of a temporal resolution and we want to retrieve the maximum of a stock using a small number of memory only polynomial rates. Our algorithm is adaptive—each query depends on previous observations actually as it is optimized. In other words, observing the function actually needs to optimize this realization with precision measuring it.

Theorem: For any $\varepsilon < 1/2$

\[
\mathbb{P}[M - W(\tilde{t}_\varepsilon) > \varepsilon] \leq \varepsilon
\]

\[
\mathbb{E}[N_\varepsilon] \leq c \log^2(1/\varepsilon)
\]

Guarantees

- Correctness: algorithm definition + the law of Brownian bridge
- 2. at OOB evaluates pretty much only near-optimal points
- Denisov (1984): rewrite the motion as two Brownian meanders
- By Durrett et al. (1977) the expected number of near-optimal points is bounded as $\mathbb{E}[X_n(0)] \leq n d^2 2^n$ which is $\mathcal{O}(\log(1/\varepsilon))$

Open problem

- Munos (2011) classifies functions according to $(d, C)$ to:
  - easy, $d = 0$, exponentially fast optimization
  - difficult, $d \geq 0$, polynomially fast optimization
- Open questions for a Brownian process:
  - what is its dimension $d$?
  - how fast can we optimize it?

Challenge: Brownian motion is a stochastic process!

Our answers that solve the open problem:

- $\forall t, \text{W}(t)$ is $t$-Lipschitz + $\exists C(t)$ s.t. Brownian $\in (d, C(t))$
- there is no $(d, C)$ with $C < \infty$ such that Brownian $\in (d, C)$
- we can optimize it with sample complexity of $\mathcal{O}(\log^2(1/\varepsilon))$