Sample-efficient Monte-Carlo planning

J. Grill, M. Valko, R. Munos

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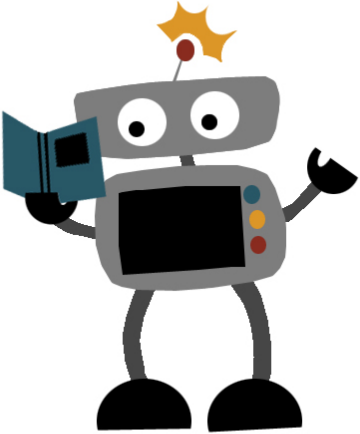
Planning in Markov Decision Processes



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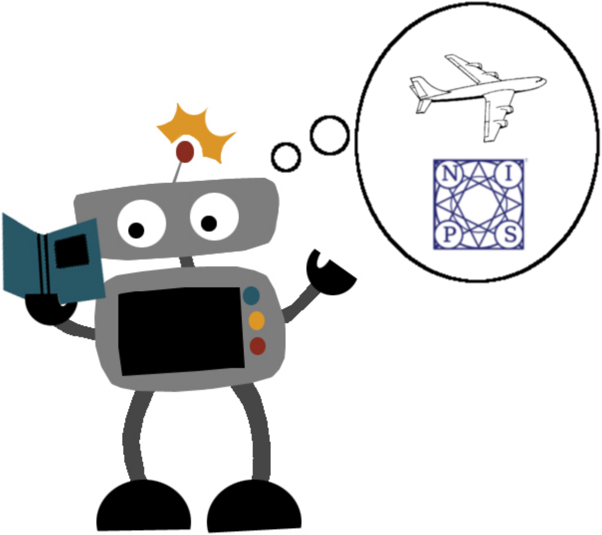

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- We have a generative model.
- We are only interested in the policy for our current environment configuration.

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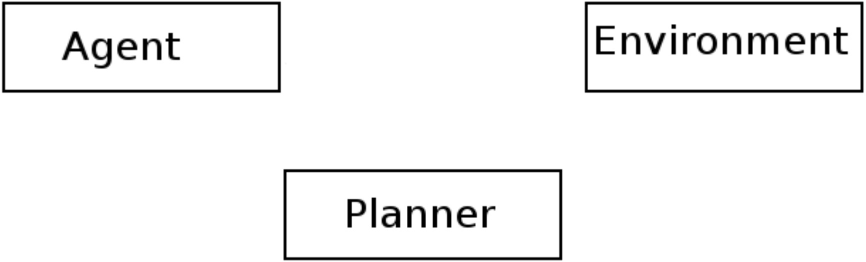

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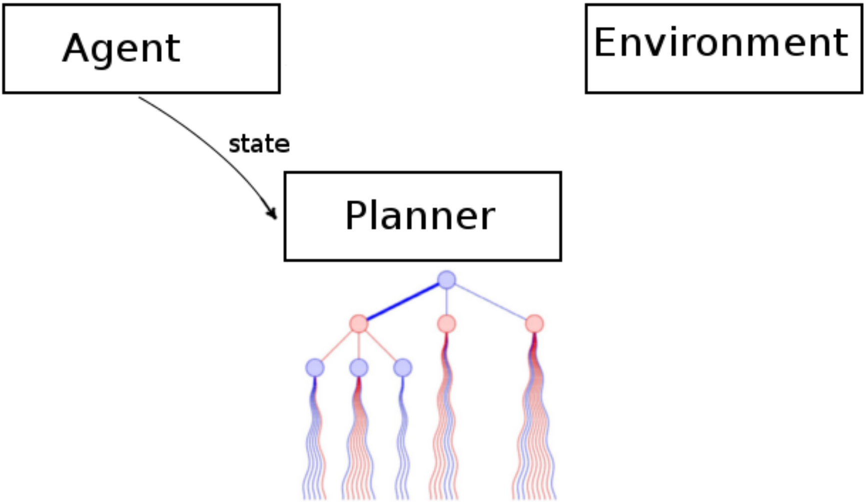
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Environment
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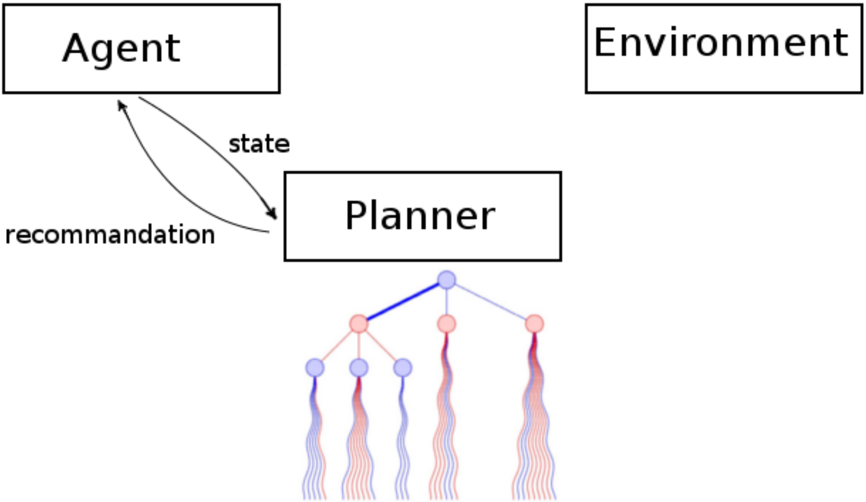

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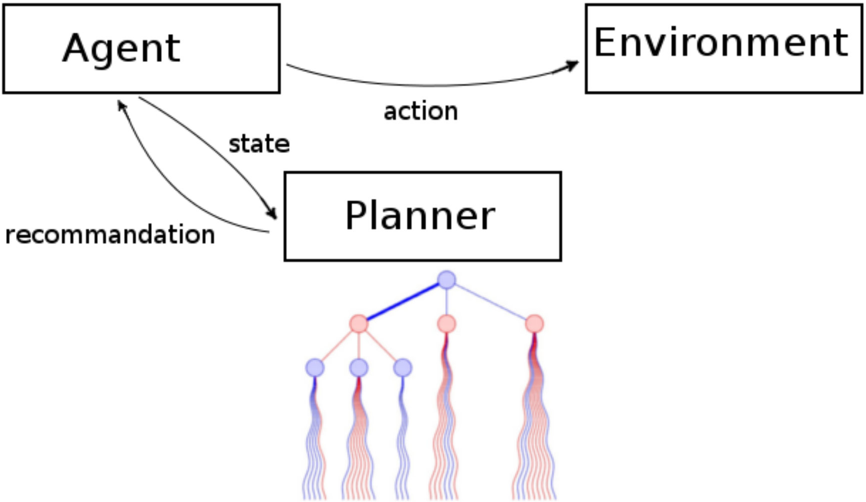
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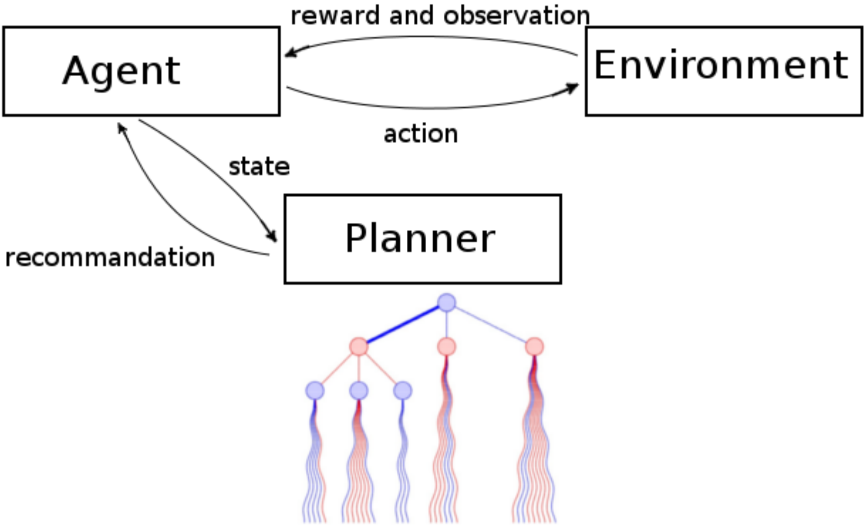
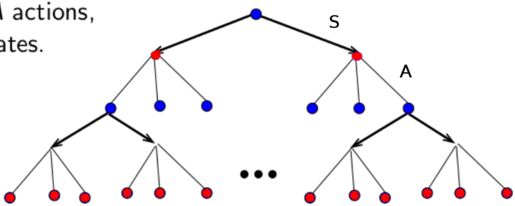


Tree representation

Assume A actions,
S next-states.



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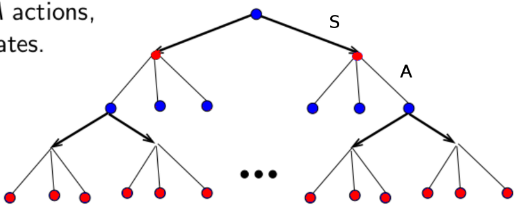

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$\mathcal{V}[s]$: discounted sum of rewards you get if you play optimally from s.

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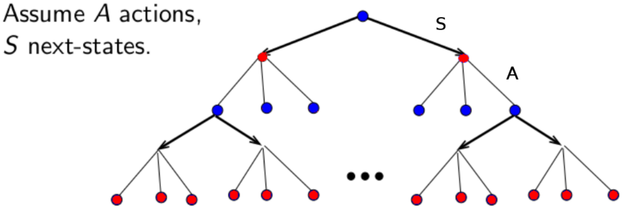

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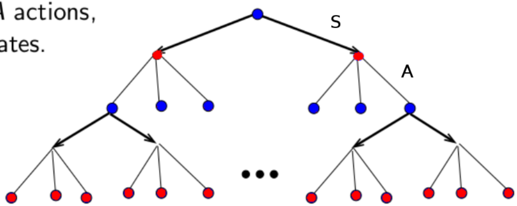

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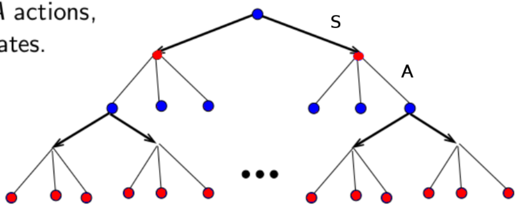

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- Average nodes (environment): $\mathcal{V}[s] = r(s) + \gamma \sum_{s' \text{ child of } s} p(s'|s)\mathcal{V}[s']$.

Goal: Compute the value of the root $\mathcal{V}[s_0]$.

Hypothesis

We assume the access to a **generative model:**

$$\text{average node } s \longrightarrow \begin{array}{ll} r_s & \text{reward sample s.t. } \mathbb{E}r = r(s) \\ y_s & \text{next state sample} \sim p(\cdot|s) \end{array}$$

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PAC (Probably Approximately Correct)

For any $\delta > 0, \epsilon > 0$, we compute $v(\delta, \epsilon)$ such that

$$\mathbb{P}\left[|v(\delta,\epsilon) - \mathcal{V}[s_0]| < \epsilon\right] > 1 - \delta$$

Adaptive Algorithm

Sample complexity: the number of calls to the generative model.

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Design **adaptive** algorithms that doesn't explore uniformly the whole tree.

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- The bias of the estimator is of order ϵ
- The variance of the estimator of order $1/n$.

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- is computationally efficient and easy to implement.
- is adaptive.

Sample complexity in the finite S case

UCT: [L. Kocsis and C. Szepesvári, 2006]

Asymptotic analysis but no finite time guarantees.

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Problem: algorithm consider the set of all policies which grows exponentially with the number of states.

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Sample complexity bound of `TrailBlazer`

$$\mathcal{O}\left((1/\epsilon)^{\max\left(2,\frac{\log(\kappa S)}{\log(1/\gamma)}\right)}\right)$$

- From $+$ to max.
- Computationally efficient.

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Adaptive planning: [Walsh et al, 2010]

Still no polynomial bound.

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When d is finite: polynomial S-independent bound.

Example of $d = 0$

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Sample complexity of order $(1/\epsilon)^2$, same as **Monte Carlo sampling**.

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- Upper bound: non-polynomial
- Lower bound: polynomial

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`TrailBlazer` can be seen as a **natural extension of Monte Carlo Sampling to control problems.**

Thank You !

Poster number: 193