Simple regret for infinitely many armed bandit

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The bandit problem considered

Simple regret for infinitely many armed bandit

- Mean reservoir distr. $F$ bounded by $\bar{\mu}^*$
- Limited sampling resources $n$

At time $t \leq n$ one can either
- sample a new arm $\nu_{K_t}$ from the reservoir distr. with mean $\mu_{K_t} \sim F$, and set $I_t = K_t$,
- or choose an arm $I_t$ among the $K_{t-1}$ observed arms $\{\nu_k\}_{k \leq K_{t-1}}$,
and then collect $X_t \sim \nu_{k_t}$

Objective: after $n$ rounds, return an arm $\hat{k}$ whose mean $\mu_{\hat{k}}$ is as large as possible. Minimize the simple regret

$$r_n = \bar{\mu}^* - \mu_{\hat{k}},$$

where $\bar{\mu}^*$ is the right end point of $1 - F$.

At time $t = 0$: 

![1 - Mean reservoir distribution](image-url)
The bandit problem considered

Simple regret for infinitely many armed bandit

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- Limited sampling resources $n$

At time $t \leq n$ one can either

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Objective: after $n$ rounds, return an arm $\hat{k}$ whose mean $\mu_{\hat{k}}$ is as large as possible. Minimize the simple regret

$$r_n = \bar{\mu}^* - \mu_{\hat{k}},$$

where $\bar{\mu}^*$ is the right end point of $1 - F$.

At time $t = 1$: 

1 - Mean reservoir distribution

Arm 1
The bandit problem considered

Simple regret for infinitely many armed bandit

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- Limited sampling resources $n$

At time $t \leq n$ one can either

- sample a new arm $\nu_{K_t}$ from the reservoir distr. with mean $\mu_{K_t} \sim F$, and set $I_t = K_t$,
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Objective: after $n$ rounds, return an arm $\hat{k}$ whose mean $\mu_{\hat{k}}$ is as large as possible. Minimize the simple regret

$$r_n = \bar{\mu}^* - \mu_{\hat{k}},$$

where $\bar{\mu}^*$ is the right end point of $1 - F$.

At time $t = 1$: 

1 - Mean reservoir distribution
The bandit problem considered

Simple regret for infinitely many armed bandit

- Mean reservoir distr. $F$ bounded by $\bar{\mu}^*$
- Limited sampling resources $n$

At time $t \leq n$ one can either

- sample a new arm $\nu_{K_t}$ from the reservoir distr. with mean $\mu_{K_t} \sim F$, and set $I_t = K_t$,
- or choose an arm $I_t$ among the $K_{t-1}$ observed arms $\{\nu_k\}_{k \leq K_{t-1}}$, and then collect $X_t \sim \nu_{k_t}$

**Objective:** after $n$ rounds, return an arm $\hat{k}$ whose mean $\mu_{\hat{k}}$ is as large as possible. Minimize the *simple regret*

$$r_n = \bar{\mu}^* - \mu_{\hat{k}},$$

where $\bar{\mu}^*$ is the right end point of $1 - F$.

At time $t = 2$:
The bandit problem considered

Simple regret for infinitely many armed bandit

- Mean reservoir distr. $F$ bounded by $\bar{\mu}^*$
- Limited sampling resources $n$

At time $t \leq n$ one can either

- sample a new arm $\nu_{K_t}$ from the reservoir distr. with mean $\mu_{K_t} \sim F$, and set $I_t = K_t$,
- or choose an arm $I_t$ among the $K_{t-1}$ observed arms $\{\nu_k\}_{k \leq K_{t-1}}$

and then collect $X_t \sim \nu_{k_t}$

**Objective:** after $n$ rounds, return an arm $\hat{k}$ whose mean $\mu_{\hat{k}}$ is as large as possible. Minimize the *simple regret*

$$r_n = \bar{\mu}^* - \mu_{\hat{k}},$$

where $\bar{\mu}^*$ is the right end point of $1 - F$.

At time $t = 2$:
The bandit problem considered

Simple regret for infinitely many armed bandit

- Mean reservoir distr. $F$ bounded by $\bar{\mu}^*$
- Limited sampling resources $n$

At time $t \leq n$ one can either
- sample a new arm $\nu_{K_t}$ from the reservoir distr. with mean $\mu_{K_t} \sim F$, and set $I_t = K_t$,
- or choose an arm $I_t$ among the $K_{t-1}$ observed arms $\{\nu_k\}_{k \leq K_{t-1}}$, and then collect $X_t \sim \nu_k$

**Objective:** after $n$ rounds, return an arm $\hat{k}$ whose mean $\mu_{\hat{k}}$ is as large as possible. Minimize the *simple regret*

$$r_n = \bar{\mu}^* - \mu_{\hat{k}},$$

where $\bar{\mu}^*$ is the right end point of $1 - F$.

At time $t = 3$:
The bandit problem considered

Simple regret for infinitely many armed bandit

- Mean reservoir distr. $F$ bounded by $\mu^*$
- Limited sampling resources $n$

At time $t \leq n$ one can either

- sample a new arm $\nu_{K_t}$ from the reservoir distr. with mean $\mu_{K_t} \sim F$, and set $I_t = K_t$,
- or choose an arm $I_t$ among the $K_{t-1}$ observed arms $\{\nu_k\}_{k \leq K_{t-1}}$, and then collect $X_t \sim \nu_{k_t}$

Objective: after $n$ rounds, return an arm $\hat{k}$ whose mean $\mu_{\hat{k}}$ is as large as possible. Minimize the simple regret

$$r_n = \bar{\mu}^* - \mu_{\hat{k}},$$

where $\bar{\mu}^*$ is the right end point of $1 - F$.

At time $t = 3$:  

1 - Mean reservoir distribution
The bandit problem considered

Simple regret for infinitely many armed bandit

- Mean reservoir distr. $F$ bounded by $\bar{\mu}^*$
- Limited sampling resources $n$

At time $t \leq n$ one can either

- sample a new arm $\nu_{K_t}$ from the reservoir distr. with mean $\mu_{K_t} \sim F$, and set $I_t = K_t$,
- or choose an arm $I_t$ among the $K_{t-1}$ observed arms $\{\nu_k\}_{k \leq K_{t-1}}$, and then collect $X_t \sim \nu_{k_t}$

Objective: after $n$ rounds, return an arm $\hat{k}$ whose mean $\mu_{\hat{k}}$ is as large as possible. Minimize the simple regret

$$r_n = \bar{\mu}^* - \mu_{\hat{k}},$$

where $\bar{\mu}^*$ is the right end point of $1 - F$.

At time $t = 4$:
The bandit problem considered

Simple regret for infinitely many armed bandit

- Mean reservoir distr. $F$ bounded by $\bar{\mu}^*$
- Limited sampling resources $n$

At time $t \leq n$ one can either

- sample a new arm $\nu_{K_t}$ from the reservoir distr. with mean $\mu_{K_t} \sim F$, and set $I_t = K_t$,
- or choose an arm $I_t$ among the $K_{t-1}$ observed arms $\{\nu_k\}_{k \leq K_{t-1}}$, and then collect $X_t \sim \nu_{k_t}$

**Objective:** after $n$ rounds, return an arm $\hat{k}$ whose mean $\mu_{\hat{k}}$ is as large as possible. Minimize the *simple regret*

$$r_n = \bar{\mu}^* - \mu_{\hat{k}},$$

where $\bar{\mu}^*$ is the right end point of $1 - F$.

At time $t = 5$: 

1 - Mean reservoir distribution

Arm 1  Arm 2  Arm 3

Arm 4
The bandit problem considered

Simple regret for infinitely many armed bandit

- Mean reservoir distr. $F$ bounded by $\bar{\mu}^*$
- Limited sampling resources $n$

At time $t \leq n$ one can either

- sample a new arm $\nu_{K_t}$ from the reservoir distr. with mean $\mu_{K_t} \sim F$, and set $I_t = K_t$,
- or choose an arm $I_t$ among the $K_{t-1}$ observed arms $\{\nu_k\}_{k \leq K_{t-1}}$, and then collect $X_t \sim \nu_{k_t}$

**Objective:** after $n$ rounds, return an arm $\hat{k}$ whose mean $\mu_{\hat{k}}$ is as large as possible. Minimize the *simple regret*

$$r_n = \bar{\mu}^* - \mu_{\hat{k}},$$

where $\bar{\mu}^*$ is the right end point of $1 - F$.

At time $t = 6$:
The bandit problem considered

Simple regret for infinitely many armed bandit

- Mean reservoir distr. $F$ bounded by $\bar{\mu}^*$
- Limited sampling resources $n$

At time $t \leq n$ one can either

- sample a new arm $\nu_{K_t}$ from the reservoir distr. with mean $\mu_{K_t} \sim F$, and set $I_t = K_t$,
- or choose an arm $I_t$ among the $K_{t-1}$ observed arms $\{\nu_k\}_{k \leq K_{t-1}}$, and then collect $X_t \sim \nu_{k_t}$

Objective: after $n$ rounds, return an arm $\hat{k}$ whose mean $\mu_{\hat{k}}$ is as large as possible. Minimize the simple regret

$$r_n = \bar{\mu}^* - \mu_{\hat{k}}$$

where $\bar{\mu}^*$ is the right end point of $1 - F$.

At time $t = 7$: 

1 - Mean reservoir distribution

Arm 1

Arm 2

Arm 3

Arm 4

Arm 5
The bandit problem considered

Simple regret for infinitely many armed bandit

- Mean reservoir distr. $F$ bounded by $\bar{\mu}^*$
- Limited sampling resources $n$

At time $t \leq n$ one can either

- sample a new arm $\nu_{K_t}$ from the reservoir distr. with mean $\mu_{K_t} \sim F$, and set $I_t = K_t$,
- or choose an arm $I_t$ among the $K_{t-1}$ observed arms $\{\nu_k\}_{k \leq K_{t-1}}$, and then collect $X_t \sim \nu_{k_t}$

**Objective:** after $n$ rounds, return an arm $\hat{k}$ whose mean $\mu_{\hat{k}}$ is as large as possible. Minimize the *simple regret*

$$r_n = \bar{\mu}^* - \mu_{\hat{k}},$$

where $\bar{\mu}^*$ is the right end point of $1 - F$.

At time $t = 8$: 

[Diagram showing mean reservoir distribution and observed arms for Arm 1, Arm 2, Arm 3, Arm 4, Arm 5]
The bandit problem considered

Simple regret for infinitely many armed bandit

- Mean reservoir distr. $F$ bounded by $\bar{\mu}^*$
- Limited sampling resources $n$

At time $t \leq n$ one can either

- sample a new arm $\nu_{K_t}$ from the reservoir distr. with mean $\mu_{K_t} \sim F$, and set $I_t = K_t$,
- or choose an arm $I_t$ among the $K_{t-1}$ observed arms $\{\nu_k\}_{k \leq K_{t-1}}$, and then collect $X_t \sim \nu_{k_t}$

Objective: after $n$ rounds, return an arm $\hat{k}$ whose mean $\mu_{\hat{k}}$ is as large as possible. Minimize the simple regret

$$r_n = \bar{\mu}^* - \mu_{\hat{k}},$$

where $\bar{\mu}^*$ is the right end point of $1 - F$.

At time $t = 9$: [Diagram with 6 arms and their distributions]
The bandit problem considered

Simple regret for infinitely many armed bandit

- Mean reservoir distr. \( F \) bounded by \( \bar{\mu}^* \)
- Limited sampling resources \( n \)

At time \( t \leq n \) one can either

- sample a new arm \( \nu_{K_t} \) from the reservoir distr. with mean \( \mu_{K_t} \sim F \), and set \( I_t = K_t \),
- or choose an arm \( I_t \) among the \( K_{t-1} \) observed arms \( \{\nu_k\}_{k \leq K_{t-1}} \), and then collect \( X_t \sim \nu_{k_t} \)

**Objective:** after \( n \) rounds, return an arm \( \hat{k} \) whose mean \( \mu_{\hat{k}} \) is as large as possible. Minimize the *simple regret*

\[
r_n = \bar{\mu}^* - \mu_{\hat{k}},
\]

where \( \bar{\mu}^* \) is the right end point of \( 1 - F \).

At time \( t \)...:
The bandit problem considered

Simple regret for infinitely many armed bandit

- Mean reservoir distr. $F$ bounded by $\bar{\mu}^*$
- Limited sampling resources $n$

At time $t \leq n$ one can either

- sample a new arm $\nu_{K_t}$ from the reservoir distr. with mean $\mu_{K_t} \sim F$, and set $I_t = K_t$,
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Objective: after $n$ rounds, return an arm $\hat{k}$ whose mean $\mu_{\hat{k}}$ is as large as possible. Minimize the simple regret

$$r_n = \bar{\mu}^* - \mu_{\hat{k}},$$

where $\bar{\mu}^*$ is the right end point of $1 - F$.

At time $t = n$: 

1 - Mean reservoir distribution

Arm 1  Arm 2  Arm 3

Arm returned

Arm 4  Arm 5  Arm 6
The bandit problem considered

**Simple regret for infinitely many armed bandit**

- Mean reservoir distr. $F$ bounded by $\mu^*$
- Limited sampling resources $n$

At time $t \leq n$ one can either

- sample a new arm $\nu_{K_t}$ from the reservoir distr. with mean $\mu_{K_t} \sim F$, and set $I_t = K_t$,
- or choose an arm $I_t$ among the $K_{t-1}$ observed arms $\{\nu_k\}_{k \leq K_{t-1}}$, and then collect $X_t \sim \nu_{k_t}$

**Objective:** after $n$ rounds, return an arm $\hat{k}$ whose mean $\mu_{\hat{k}}$ is as large as possible. Minimize the *simple regret* $r_n = \bar{\mu}^* - \mu_{\hat{k}}$,

where $\bar{\mu}^*$ is the right end point of $1 - F$.

**Double exploration dilemma here:** Allocation both to (i) learn the characteristics of the arm reservoir distr. (*meta-exploration*) and (ii) learn the characteristics of the arms (*exploration*).

**Main questions**

How many arms should be sampled from the arm reservoir distribution? How aggressively should these arms be explored?
Applications

Simple-regret bandit problems with a *large number of arms* or with a *small budget*:
- Selection of a good biomarker
- Special case of *feature selection* where one wants to select a single feature [Hauskrecht et al., 2006]
Literature review

- **Simple regret bandits:** [Even-Dar et al., 2006], [Audibert et al., 2010], [Kalyanakrishnan et al., 2012], [Kaufmann et al., 2013], [Karnin et al., 2013], [Gabillon et al., 2012], and [Jamieson et al., 2014]

- **Infinitely many armed bandits with cumulative regret:** [Berry et al., 1997], [Wang et al., 2008], and [Bonald and Proutière, 2013].

- **Infinitely many armed settings with arm structure:** [Dani et al., 2008], [Kleinberg et al., 2008], [Munos, 2014], [Azar et al., 2014]
Literature review

- **Simple regret bandits:**
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- **Infinitely many armed settings with arm structure:** [Dani et al., 2008], [Kleinberg et al., 2008], [Munos, 2014], [Azar et al., 2014]

**Results:**

- offer strategies that provide an optimal (or $\epsilon$-optimal) arm with high probability.

- provide stopping rule based strategies that sample until they can provide an $\epsilon$-optimal arm.

**But:**

- Fixed number of arms that is smaller than the budget $n$ (importance of trying each arm).
Literature review

- **Simple regret bandits:** [Even-Dar et al., 2006], [Audibert et al., 2010], [Kalyanakrishnan et al., 2012], [Kaufmann et al., 2013], [Karnin et al., 2013], [Gabillon et al., 2012], and [Jamieson et al., 2014]

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- **Infinitely many armed settings with arm structure:** [Dani et al., 2008], [Kleinberg et al., 2008], [Munos, 2014], [Azar et al., 2014]

Results:

- Provide optimal strategies under shape constraint on $F$ and boundedness of the arm distributions.

But:

- Cumulative regret.

Note

We will discuss this in details soon...
Literature review

- **Simple regret bandits:** [Even-Dar et al., 2006], [Audibert et al., 2010], [Kalyanakrishnan et al., 2012], [Kaufmann et al., 2013], [Karnin et al., 2013], [Gabillon et al., 2012], and [Jamieson et al., 2014]

- **Infinitely many armed bandits with cumulative regret:** [Berry et al., 1997], [Wang et al., 2008], and [Bonald and Proutière, 2013]

- **Infinitely many armed settings with arm structure:** [Dani et al., 2008], [Kleinberg et al., 2008], [Munos, 2014], [Azar et al., 2014]

**Results:**

- Provide optimal strategies for specific structured bandits.

**But:**

- Structure or contextual information needed.
Back on infinitely many armed bandits literature

**IMAB with cumulative regret:** [Berry et al., 1997], [Wang et al., 2008], and [Bonald and Proutière, 2013].

**Cumulative regret:**

\[ R_n^C = n\bar{\mu}^* - \sum_{t \leq n} X_t. \]

**Crucial assumption:**

\[ \mathbb{P}_{\mu \sim F} (\bar{\mu}^* - \mu \geq \varepsilon) \approx \varepsilon^\beta, \]

i.e. \( 1 - F \) is \( \beta \)-regularly varying in \( \bar{\mu}^* \).
Back on infinitely many armed bandits literature

**IMAB with cumulative regret:** [Berry et al., 1997], [Wang et al., 2008], and [Bonald and Proutière, 2013].

**Cumulative regret:**

\[ R_n^C = n\bar{\mu}^* - \sum_{t \leq n} X_t. \]

**Crucial assumption:**

\[ P_{\mu \sim F} (\bar{\mu}^* - \mu \geq \varepsilon) \approx \varepsilon^\beta, \]

i.e. $1 - F$ is $\beta$-regularly varying in $\bar{\mu}^*$.

**Requirements:** Bounded arm distributions and *knowledge of $\beta$ for choosing the nb. of arms.*

**Theorem (Regret bound)**

Minimax bound on $\mathbb{E}(R_n^C)$

\[ \mathcal{O} \left( \max \left( n^{\frac{\beta}{\beta + 1}}, \sqrt{n} \right) \right) \text{ up to } \log(n). \]

**Special case:** If arm distr. bounded by $\bar{\mu}^*$, different rate.

**Theorem (Special regret)**

Minimax bound on $\mathbb{E}(R_n^C)$

\[ \mathcal{O} \left( n^{\frac{\beta}{\beta + 1}} \right) \text{ up to } \log(n). \]
The simple regret setting and assumptions

**Objective:**

Minimize the simple regret in the infinitely many armed setting

\[ r_n = \bar{\mu}^* - \mu_{\hat{k}}. \]

Same assumptions as for IMAB with cumulative regret:

- Regularly varying mean reservoir distr.:
  \[ \mathbb{P}_{\mu \sim F} (\bar{\mu}^* - \mu \geq \varepsilon) \approx \varepsilon^\beta \]

- Distributions from the arms are bounded/sub-Gaussian.
Lower bound

The following lower bound holds.

**Theorem (CV15)**

The expected simple regret $\mathbb{E}(r_n)$ can be lower bounded as

$$\max \left( n^{-\frac{1}{\beta}}, n^{-1/2} \right).$$

**Remark:** Different bottleneck as for the cumulative regret

$$\mathbb{E}[R_n^C] = \mathcal{O} \left( \max \left( n^{\frac{\beta}{\beta+1}}, \sqrt{n} \right) \right).$$

Strategy that attains this bound?
The SiRI strategy

Parameters: $\beta, C, \delta$.  
Pick $T_\beta \approx n^{\min(\beta,2)/2}$ arms from the reservoir  
Pull each of $T_\beta$ arms once and set $t \leftarrow T_\beta$.  
while $t \leq n$ do  
  For any $k \leq T_\beta$, set  
  $B_{k,t} \leftarrow \hat{\mu}_{k,t} + 2\sqrt{\frac{C}{T_{k,t}} \log \left( \frac{n\delta}{T_{k,t}} \right)} + \frac{2C}{T_{k,t}} \log \left( \frac{n\delta}{T_{k,t}} \right)$  
  Pull $T_{k_t,t}$ times the arm $k_t$ that maximizes the $B_{k,t}$  
  Set $t \leftarrow t + T_{k_t,t}$.  
end while  
Output: Return the most pulled arm $\hat{k}$. 
The SiRI strategy

Parameters: $\beta, C, \delta$. 

Pick $\bar{T}_\beta \approx n^{\min(\beta, 2)/2}$ arms from the reservoir 

Pull each of $\bar{T}_\beta$ arms once and set $t \leftarrow \bar{T}_\beta$. 

while $t \leq n$ do 

For any $k \leq \bar{T}_\beta$, set 

$$B_{k,t} \leftarrow \hat{\mu}_{k,t} + 2\sqrt{\frac{C}{T_{k,t}} \log \left( \frac{n\delta}{T_{k,t}} \right)} + \frac{2C}{T_{k,t}} \log \left( \frac{n\delta}{T_{k,t}} \right)$$

Pull $T_{k,t,t}$ times the arm $k_t$ that maximizes the $B_{k,t}$ 

Set $t \leftarrow t + T_{k,t,t}$.

end while 

Output: Return the most pulled arm $\hat{k}$.
The SiRI strategy

**Parameters:** $\beta, C, \delta$.

Pick $\bar{T}_\beta \approx n^{\min(\beta, 2)/2}$ arms from the reservoir.

Pull each of $\bar{T}_\beta$ arms once and set $t \leftarrow \bar{T}_\beta$.

**while** $t \leq n$ **do**

For any $k \leq \bar{T}_\beta$, set

$$B_{k,t} \leftarrow \hat{\mu}_{k,t} + 2\sqrt{\frac{C}{T_{k,t}}} \log \left( \frac{n\delta}{T_{k,t}} \right) + \frac{2C}{T_{k,t}} \log \left( \frac{n\delta}{T_{k,t}} \right)$$

Pull $T_{k_t,t}$ times the arm $k_t$ that maximizes the $B_{k,t}$

Set $t \leftarrow t + T_{k_t,t}$.

end while

**Output:** Return the most pulled arm $\hat{k}$.

**Remark:** SiRI is the combination of a choice of the number of arms and a UCB algorithm for *cumulative* regret.
The following upper bound holds.

**Theorem (CV15)**

The expected simple regret $\mathbb{E}(r_n)$ of SiRI can be upper bounded up to $\log(n)$ factors as

$$\max \left( n^{-1/2}, n^{-\frac{1}{\beta}} \right).$$

Lower and upper bound match up to $\log(n)$ factors (not present in all cases).
Extensions

In the paper we present three main extensions:

- **anytime SiRI.**

- **Distributions bounded by $\mu^*$**: A Bernstein modification of SiRI has Minimax optimal simple regret

$$\max\left(n^{-1}, n^{-1/\beta}\right).$$

- **Unknown $\beta$**: Possible to estimate $\beta$ using arguments from extreme value theory. Simple regret rate is the same up to $\log(n)$ factors. Same could apply to cumulative regret.
Recap on the rates (up to $\log(n)$)

<table>
<thead>
<tr>
<th></th>
<th>Minimax optimal rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cumulative regret</td>
<td>$\max\left(n^{\frac{\beta}{\beta+1}}, \sqrt{n}\right)$</td>
</tr>
<tr>
<td>Cum. regret with arm bound $\bar{\mu}^*$</td>
<td>$n^{\frac{\beta}{\beta+1}}$</td>
</tr>
<tr>
<td>Simple regret</td>
<td>$\max\left(n^{-1/\beta}, n^{-1/2}\right)$</td>
</tr>
<tr>
<td>Simple regret with arm bound $\bar{\mu}^*$</td>
<td>$\max\left(n^{-1/\beta}, n^{-1}\right)$</td>
</tr>
</tbody>
</table>

**Remark:** Different bottleneck as for the cumulative regret.
Simulations

Comparison on synthetic data of SiRI with:

- lil’UCB [Jamieson et. al, 2014], to which the optimal oracle number of arms is given (algorithm for simple regret with finitely many arms)
- UCB-F of [Wang et. al, 2008] (algorithm for cumulative regret and infinitely many arms)
Figure: Comparison on $B(1, 1)$ (UL), $B(1, 2)$ (UR), and $B(1, 3)$ (DL), and unknown $\beta$ on $B(1, 1)$ (DR)
Conclusion

Minimax optimal solution up to $\log(n)$ factors for the simple regret problem with infinitely many arms. Extensions:

- Unknown $\beta$

- Bernstein SiRI with minimax optimal performance when arm distributions are bounded by $\bar{\mu}^*$

Open problems:

- Closing the log gaps (some of them are already closed)?
- Heavy tailed mean reservoir distribution?

THANK YOU!

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