

# Functional programming in R (R'20)

## TD 1: Higher-order functions and their types

In the following exercises we assume the following built-in functions with the corresponding types:

$\text{sum} : \text{num}^* \rightarrow \text{num}$	$\text{mean} : \text{num}^* \rightarrow \text{num}$	$\text{length} : \mathcal{X}^* \rightarrow \text{int}$
$\text{any} : \text{log}^* \rightarrow \text{log}$	$\text{all} : \text{log}^* \rightarrow \text{log}$	
$\text{substr} : \text{chr} \times \text{int} \times \text{int} \rightarrow \text{chr}$	$\text{paste} : \text{chr} \times \text{chr} \rightarrow \text{chr}$	$\text{ceil} : \text{num} \rightarrow \text{int}$
$'*': \text{num} \times \text{num} \rightarrow \text{num}$	$'+': \text{num} \times \text{num} \rightarrow \text{num}$	$'-': \text{num} \times \text{num} \rightarrow \text{num}$

**Exercise 1.** Define the (most general) type of the following expressions

1. `function (x,y) <x+y,x*y>`
2. `function (s,x,y) <substr(s,x,y),y-x+1>`
3. `function (v) <any(v),all(v)>`
4. `function (f,v) <any(v),f(v)>`
5. `function (f,g,s) substr(s,f(s),g(s))`
6. `function (f,g,x) <f(x),g(x)>`
7. `function (f,v) f(length(v)) + 1`
8. `function (f,v) length(f(v)) + 1`
9. `function (f,v) length(f(v+1))`
10. `function (v,f) <sum(v),mean(v),f(v)>`
11. `function (f,v) <sum(v),mean(v),f(v)>`
12. `function (f,g) function (v) <mean(f(v)),all(g(v)),g(v),f(v)>`
13. `function (f,g,x) f(g(x)+1)`
14. `function (f,g,s) paste(f(s),g(s))`
15. `function (f,x,y,z) paste(paste(f(x+1),f(y+2)),f(z+3))`
16. `function (f,x,y,z) paste(paste(f(x),f(y)),f(z))`
17. `function (f,g,s) <ceil(f(s)),paste(s,g(s))>`

**Exercise 2.** Propose the (most general) type for the following functions:

1. `function (F,g,h,x) F(g(h(x)))`
2. `function (f,x) <paste("Hello",f(x)),substr(f(x),0,3)>`
3. `function (f,x) <any(x),length(x),f(x)>`
4. `function (f,x) (function (y) y+1) (f(x))`
5. `function (F,g,x) (function (h) h(x))(F(g))`
6. `function (F,g,x) F(g)(x)`
7. `function (f,x,y,z) <f(x),f(y)+f(z)>`
8. `function (f,x,y,z) f(x,f(y,z))`
9. `function (F,g,x) <F(g)(x),g(x)>`
10. `function (F,G,f,g,x) <F(f)(x),G(f,g)(x),f(g(x))>`

**Exercise 3.** Define the (most general) type of the following expressions

1. `function (x) function (y) <x+y,x*y>`
2. `function (s) function (x,y) <substr(s,x,y),y-x+1>`
3. `function (v) <any(v),all(v)>`
4. `function (f) function (v) <any(v),f(v)>`
5. `(function (f) function (v) <any(v),f(v)>)(all)`
6. `function (f) function (g) function (s) substr(s,f(s),g(s))`
7. `function (f) function (s,i,j) <substr(s,f(i),f(j)),i+j>`
8. `function (f,g) function (x) <f(x),g(x)>`
9. `function (f) function (v) f(length(v)) + 1`
10. `function (f) function (v) length(f(v)) + 1`
11. `function (f) function (v) length(f(v+1))`
12. `function (v) function (f) <sum(v),mean(v),f(v)>`
13. `function (f) function (v) <sum(v),mean(v),f(v)>`
14. `function (f) function (g) function (v) <mean(f(v)),all(g(v)),g(v),f(v)>`
15. `function (f) function (g) function (x) f(g(x)+1)`
16. `function (f) function (g) function (s) paste(f(s),g(s))`
17. `function (f) function (x,y,z) paste(paste(f(x+1),f(y+2)),f(z+3))`
18. `function (f) function (x,y,z) paste(paste(f(x),f(y)),f(z))`
19. `function (f) function (g) function (s) <ceil(f(s)),paste(s,g(s))>`
20. `function (f,g) function (x,y) <mean(f(x)),g(x)+g(y)>`

**Exercise 4.** Propose the (most general) type for the following functions:

1. `function (F) function (g) function (h) function (x) F(g(h(x)))`
2. `function (f) function (x) <paste("Hello",f(x)),substr(f(x),0,3)>`
3. `function (f) function (g) function (x) <f(x),g(x)>`
4. `function (F) function (g) function (x) (function (h) h(x))(F(g))`
5. `function (F) function (g) function (x) F(g)(x)`
6. `function (f) function (x,y,z) <f(x),f(y),f(z)>`
7. `function (f) function (x,y,z) f(x,f(y,z))`
8. `function (F) function (g) function (x) <F(g)(x),g(x)>`
9. `function (F,G) function (f,g) function (x) <F(f)(x),G(f,g)(x),f(g(x))>`