



R: Control and data flow

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2020

Outline



Expressions

Function calls

Branching expression (if/else)

Loops

Scope and evaluation

Expressions

Functional programming paradigm

- ▶ Program is an expression \mathcal{E}
- ▶ Running the program = evaluating the expression \mathcal{E}

Basic expression building blocks

- ▶ **Assignment** $x \leftarrow \mathcal{E}$ assigns a value to the variable and evaluates to the value of x
- ▶ **Function application** $f(\mathcal{E}_1, \dots, \mathcal{E}_n)$ calls the function and evaluates to the result returned by the function
- ▶ **Composition** $\mathcal{E}_0; \mathcal{E}_1$ evaluates to the value of the last expression \mathcal{E}_1
- ▶ **Grouping** $\{ \mathcal{E} \}$ evaluates to the inner expression (for structuring purposes)

Example

► $y \leftarrow x \leftarrow 1 \mapsto$

Example

▶ $y \leftarrow x \leftarrow 1 \mapsto 1$ $x = 1$ and $y = 1$

▶ $y \leftarrow 1 + (x \leftarrow 1) \mapsto$

Example

- ▶ $y \leftarrow x \leftarrow 1 \mapsto 1 \quad x = 1 \text{ and } y = 1$
- ▶ $y \leftarrow 1 + (x \leftarrow 1) \mapsto 2 \quad x = 1 \text{ and } y = 2$
- ▶ $x \leftarrow 1; 0 \mapsto$

Example

- ▶ $y \leftarrow x \leftarrow 1 \mapsto 1 \quad x = 1 \text{ and } y = 1$
- ▶ $y \leftarrow 1 + (x \leftarrow 1) \mapsto 2 \quad x = 1 \text{ and } y = 2$
- ▶ $x \leftarrow 1; 0 \mapsto 0 \quad x = 1$
- ▶ $y \leftarrow \{x \leftarrow 1; 0\} \mapsto$

Example

- ▶ $y \leftarrow x \leftarrow 1 \mapsto 1 \quad x = 1 \text{ and } y = 1$
- ▶ $y \leftarrow 1 + (x \leftarrow 1) \mapsto 2 \quad x = 1 \text{ and } y = 2$
- ▶ $x \leftarrow 1; 0 \mapsto 0 \quad x = 1$
- ▶ $y \leftarrow \{x \leftarrow 1; 0\} \mapsto 0 \quad x = 1 \text{ and } y = 0$
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Example

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- ▶ $x \leftarrow 1; 0 \mapsto 0$ $x = 1$
- ▶ $y \leftarrow \{x \leftarrow 1; 0\} \mapsto 0$ $x = 1$ and $y = 0$
- ▶ $y \leftarrow \{x \leftarrow \{1; 0\}\} \mapsto 0$ $x = 0$ and $y = 0$
- ▶ $2 + 3 \mapsto '+'(2, 3) \mapsto 5$
- ▶ $\text{substr}(s \leftarrow "abcde", i \leftarrow \text{nchar}(s) - 2, i + 2) \mapsto$

Example

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- ▶ $y \leftarrow \{x \leftarrow \{1; 0\}\} \mapsto 0$ $x = 0$ and $y = 0$
- ▶ $2 + 3 \mapsto '+'(2, 3) \mapsto 5$
- ▶ $\text{substr}(s \leftarrow "abcde", i \leftarrow \text{nchar}(s) - 2, i + 2) \mapsto "cde"$
 $s = "abcde"$ and $i = 3$



Function calls

Positional and named arguments



- ▶ `f ← function (a,b) a + 3*b`
- ▶ `f(10,2) ↦ 16`
- ▶ `f(2,10) ↦ 32`
- ▶ `f(a=10,b=2) ↦ 16`
- ▶ `f(b=2,a=10) ↦ 16`

- ▶ `f ← function (a,b=1,c=3) {
 if (b == 1) a + c else b + c
}`
- ▶ `f(10) ↦ 13`
- ▶ `f(10,1) ↦ 13`
- ▶ `f(10,2) ↦ 5`
- ▶ `f(10,2,4) ↦ 6`
- ▶ `f(10,c=4) ↦ 14`
- ▶ `f(b=2,c=4) ↦ 6`
- ▶ `f(b=1) ↦ error`

Ellipsis ...

Additional arguments of a function that can be passed on

Example

```
sapply ← function(v,f,...) {  
  res ← vector()  
  for (i in 1:length(v)) {  
    res[i] ← f(v[i],...)  
  }  
  res  
}
```

```
sapply(1:4,function(x,y) x+y, 2) ↦ 3 4 5 6
```

```
sapply(1:4,'+',2) ↦ 3 4 5 6
```



Branching expression (if/else)

Conditional expression (if/else)



if (\mathcal{E}) \mathcal{E}_1 else \mathcal{E}_2

- ▶ if \mathcal{E} evaluates to TRUE, evaluate and return the value of \mathcal{E}_1 ; otherwise evaluate and return the value of \mathcal{E}_2
- ▶ \mathcal{E} must be interpretable as logical
- ▶ the else part is optional; when missing and the condition \mathcal{E} is not satisfied, the whole expression evaluates to NULL

Example

- ▶ if (c(-1,1) > 0) "+" else "-" \mapsto "-"
- ▶ if (any(c(-1,1) > 0)) "+" else "-" \mapsto "+"
- ▶ if (0) "a" else "b" \mapsto "b"
- ▶ if (-2) "a" else "b" \mapsto "a"
- ▶ if ("FALSE") 1 else 0 \mapsto 0
- ▶ if ("a") 1 else 0 \mapsto error

`ifelse(test, yes, no)`

- ▶ `test` an object which can be coerced to logical
- ▶ `yes` return values for true elements of `test`
- ▶ `no` return values for false elements of `test`

Example

- ▶ `ifelse(-2:2 < 0, "-", "+")` \mapsto "-" "-" "+" "+" "+"
- ▶ `ifelse(1:4 %% 2 == 0, "E", "O")` \mapsto "O" "E" "O" "E"

Vectorized if/else (contd.)



Common misconception

```
ifelse(test, yes, no)
```

is supposed to be equivalent to

```
{tmp←yes; tmp[!test]←no; tmp}
```

However, consider

- ▶ `x ← c(1,2,3,4,6,7,8,9)`
- ▶ `x %% 2 == 0` \mapsto F T F T T F T F
- ▶ `ifelse(x %% 2 == 0, c("E","e"), c("O","o","."))`
 \mapsto "O" "e" "." "e" "E" "." "E" "o"
- ▶ `{tmp←c("E","e"); tmp[x%%2!=0]←c("O","o","."); tmp}`
 \mapsto "O" "e" "o" NA NA "." NA "O"

Switch statement



```
switch(x, "a", "b", "c")  
if (x == 1) {  
    "a"  
}  
else if (x == 2) {  
    "b"  
}  
else if (x == 3) {  
    "c"  
}  
else {  
    NULL  
}
```

```
switch(s, a=1, b=2, c=3, 4)  
if (s == "a") {  
    1  
}  
else if (s == "b") {  
    2  
}  
else if (s == "c") {  
    3  
}  
else {  
    4  
}
```



Loops



Two main looping mechanisms

- ▶ Imperative: `for`, `while`, and `repeat`
- ▶ Declarative: `apply` function family

Kinds of loops

- ▶ `for (var in C) E` iterates over elements of a collection
- ▶ `while (cond) E` iterated as long as a given condition is satisfied
- ▶ `repeat E` iterates indefinitely (unless `break` is used)
- ▶ all loops evaluate to `NULL`

Flow control inside loops

- ▶ `next` interrupts the current iteration and control flow moves to the next to next one
- ▶ `break` interrupts the execution and exits the inner most loop

For loop



`for (x in C) E`

- ▶ \mathcal{C} is a vector or list (hence also factor and data frame)
- ▶ \mathcal{E} is evaluated for x being assigned consecutive values in \mathcal{C}
- ▶ side-effect: after having finished the execution the variable x is defined and carries the last assigned value

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Example

```
sum ← function (v) {  
  acc ← 0;  
  for (x in v)  
    acc ← acc + x;  
  return(acc)  
}
```

`sum(c(1,4,2,6,1))` \mapsto 14

For loop



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Example

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sum ← function (v) {  
  acc ← 0  
  for (x in v)  
    acc ← acc + x  
  acc  
}
```

`sum(c(1,4,2,6,1))` \mapsto 14

For loop (example)



Example

```
f ← function (v) {  
  w ← numeric(length(v))  
  for (i in 1:length(v)) {  
    w[i] ← 2*v[i] + i  
  }  
  w  
}
```

$f(c(1,4,7)) \mapsto 3 \ 10 \ 17 \equiv 2*c(1,4,7) + 1:3$

For loop (examples)



Example

```
find_elem ← function (v,x) {  
  for (i in 1:length(v)) {  
    if (v[i] == x) {  
      return(TRUE)  
    }  
  }  
  FALSE  
}  
find_elem(c(1,4,7,10,3,2,1,4),2) ↦ TRUE  
find_elem(v,x) ≡ any(v == x)
```

For loop (examples)



Example

```
find_pos ← function (v,x) {  
  for (i in 1:length(v)) {  
    if (v[i] == x) {  
      return(i)  
    }  
  }  
}
```

```
find_pos(c(1,4,7,10,3,2,1,4),2) ↦ 6
```

```
find_pos(v,x) ≡ (1:length(v))[v == x][1]
```

For loop (example)



Example

```
sapply ← function(v,f) {  
  res ← vector()  
  for (i in 1:length(v)) {  
    res[i] ← f(v[i])  
  }  
  res  
}
```

`sapply(1:4,function (x) x^2)` \mapsto 1 4 9 16

`sapply(1:4,as.character)` \mapsto "1" "2" "3" "4"

`sapply(1:4,function (x) x/2.0)` \mapsto 0.5 1.0 1.5 2.0

While loop



```
while (cond)  $\mathcal{E}$ 
```

- ▶ execute \mathcal{E} again and again as long as cond evaluates to TRUE

While loop (example)



Example

```
create_polynomial ← function (p) function (x) {  
  y ← 0  
  i ← length(p)  
  while (i > 0) {  
    y ← y + p[i] * (x^(length(p)-i))  
    i ← i - 1  
  }  
  y  
}
```

```
p ← create_polynomial(c(5,4,2,3))
```

```
p(x) =  $5x^3 + 4x^2 + 2x + 3$ 
```

```
p(1) ↦ 14
```

```
p(-1) ↦ 0
```

```
p(2) ↦ 63
```


Repeat loop



repeat \mathcal{E}

- ▶ execute \mathcal{E} again and again until break is called

Repeat loop (example)



Example

```
find_root ← function(p, x1, x2) {  
  repeat {  
    y1 ← p(x1)  
    y2 ← p(x2)  
    xz ← (x1+x2)/2  
    yz ← p(xz)  
    if (sign(y1) == sign(yz))  
      x1 ← xz  
    if (sign(y2) == sign(yz))  
      x2 ← xz  
    if (abs(yz) < 10e-10)  
      break  
  }  
  xz  
}  
find_root(create_polynomial(c(5,4,2,3),-10,10)) ↦ -1
```



Apply family of functions

- ▶ `sapply` operates on vectors
- ▶ `lapply` operates on list
- ▶ `apply` operates on matrices
- ▶ `mapply` operates on multiple vectors

Ellipsis

Additional arguments that are passed through to other functions.

Example

```
sapply ← function(v,f,...) {  
  res ← vector()  
  for (i in 1:length(v)) {  
    res[i] ← f(v[i],...)  
  }  
  res  
}
```

```
sapply(1:4,function(x,y) x+y, 2) ↦ 3 4 5 6  
sapply(1:4,'+',2) ↦ 3 4 5 6
```

Most functions in R are vectorised

- ▶ when given a vector, the function is applied on every element
- ▶ `sqrt : num* → num*`
- ▶ `sqrt(c(1,2,3))` \mapsto 1.000000 1.414214 1.732051

Non-vectorised function can be vectorised with `sapply`

- ▶

```
loop ← function (n) {  
  x ← 1  
  for (i in 1:n)  
    x ← sin(x)  
  x  
}
```

- ▶ `loop(1)` \mapsto 0.8414710
- ▶ `loop(2)` \mapsto 0.7456241
- ▶ `loop(c(1,2))` \mapsto **error**
- ▶ `loop_vect ← function(v) sapply(v,loop)`
- ▶ `loop_vect(c(1,2))` \mapsto 0.8414710 0.7456241

`sapply(v,f)` will return a matrix

- ▶ when `f` returns a vector of the same length each time

Example

- ▶ `f ← function (n) cos(seq(n, (n+1), 0.5))`

- ▶ `f(1) ↦ 0.54030 0.07074 -0.41615`

- ▶ `f(2) ↦ -0.4161 -0.8011 -0.9900`

- ▶ `sapply(1:4,f)`

↓

$$\begin{bmatrix} 0.54030 & -0.4161 & -0.9900 & -0.6536 \\ 0.07074 & -0.8011 & -0.9365 & -0.2108 \\ -0.41615 & -0.9900 & -0.6536 & 0.2837 \end{bmatrix}$$

`apply(M,dim,f,...)`

- ▶ if `dim = 1`, then `f` is called on every row
- ▶ if `dim = 2`, then `f` is called on every column
- ▶ if `dim = c(1,2)`, then `f` is called on every cell

Example

- ▶ $m \leftarrow \begin{bmatrix} 1 & 4 & 9 \\ 4 & 16 & 25 \end{bmatrix}$
- ▶ `apply(m,1,sum)` \mapsto 14 45
- ▶ `apply(m,2,sum)` \mapsto 5 20 34
- ▶ `apply(m,c(1,2),sqrt)` $\mapsto \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}$

`mapply(f, v1, v2, ..., vk, ...)`

- ▶ constructs a vector v such that
- ▶ $v[i] \leftarrow f(v_1[i], v_2[i], \dots, v_k[i], \dots)$
- ▶ v is as long as the longest of the input vectors (recycling is applied if lengths are not all equal)

Example

- ▶ `mapply(function(x,y) x+y, c(1,3), c(4,6))` \mapsto 5 9
 \equiv `mapply('+', c(1,3), c(4,6))` \equiv `c(1,3) + c(4,6)`
- ▶

```
f ← function(x,y) {  
  for(i in 1:y)  
    x ← sqrt(x+1)  
  x  
}
```
- ▶ `mapply(f, 1:3, c(1,5,10))` \mapsto 1.4142 1.6191 1.6180

Scope and evaluation

Scoping

- ▶ what a "*part of a program*" means?
- ▶ variable: where is its value stored

Two types of scoping

lexical depends on the location in the source code, where the variable is defined

dynamic depends on the execution context



- ▶ *environment* is a frame of reference for variable lookup
- ▶ organized into a *stack* (but actually a *tree*)
- ▶ if a variable doesn't exist in the current environment, then check its parent, then its grandparent, etc.
- ▶ *global* environment, storing all global variables, is at the end of the search path (*root* environment)
- ▶ when function is called a *new environment* is created, function parameters are new variables in the new environment
- ▶ the parent environment of a function call is always the environment where the function has been *defined* (and **not** where the function has been called)

What are the environments?



► $x \leftarrow 1$

$x = 1$

What are the environments?



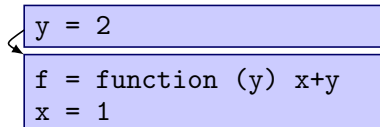
- ▶ `x ← 1`
- ▶ `f ← function (y) x + y`

```
f = function (y) x+y  
x = 1
```

What are the environments?



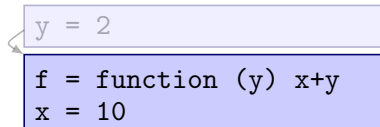
- ▶ `x ← 1`
- ▶ `f ← function (y) x + y`
- ▶ `f(2) ↦ 3`



What are the environments?



- ▶ `x ← 1`
- ▶ `f ← function (y) x + y`
- ▶ `f(2) ↦ 3`
- ▶ `x ← 10`

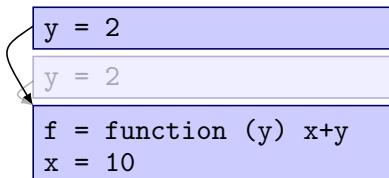


What are the environments?



- ▶ `x ← 1`
- ▶ `f ← function (y) x + y`
- ▶ `f(2) ↦ 3`

- ▶ `x ← 10`
- ▶ `f(2) ↦ 12`



What are the environments?



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What are the environments?



- ▶ `x ← 1`
- ▶ `f ← function (y) x + y`

```
f = function (y) x+y  
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```

What are the environments?



```
▶ x ← 1
▶ f ← function (y) x + y
▶ g ← function (y) {
  x ← 10
  x + y
}
```

```
g = function (y) {...}
f = function (y) x+y
x = 1
```

What are the environments?



- ▶ `x ← 1`
- ▶ `f ← function (y) x + y`
- ▶ `g ← function (y) {`
 `x ← 10`
 `x + y`
}
- ▶ `f(1) ↦ 2`

```
y = 1
g = function (y) {...}
f = function (y) x+y
x = 1
```

What are the environments?

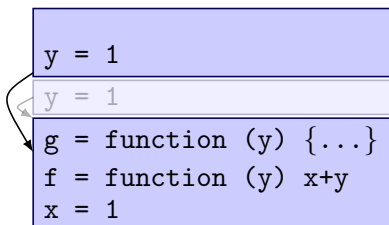


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 `x ← 10`
 `x + y`
}
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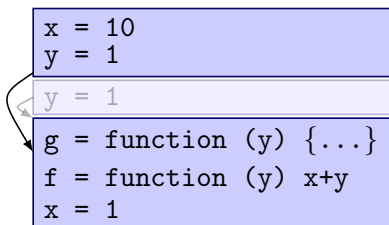
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- ▶ `g ← function (y) {
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}`
- ▶ `f(1) ↦ 2`
- ▶ `g(1) ↦ 11`



What are the environments?



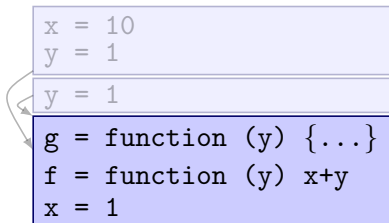
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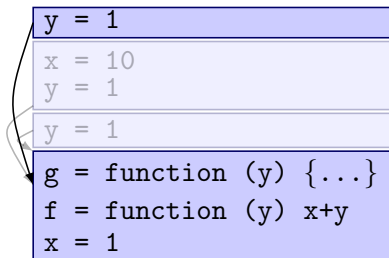


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```
f = function (y) x+y  
x = 1
```

What are the environments?



```
▶ x ← 1
▶ f ← function (y) x + y
▶ h ← function (y) {
  x ← 10
  x + y
}
```

```
h = function (y) {...}
f = function (y) x+y
x = 1
```

What are the environments?



- ▶ `x ← 1`
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h = function (y) {...}  
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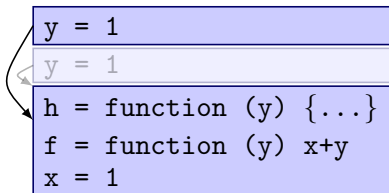
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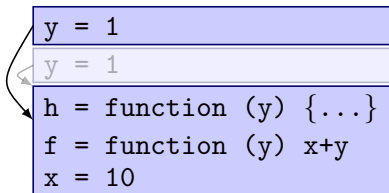
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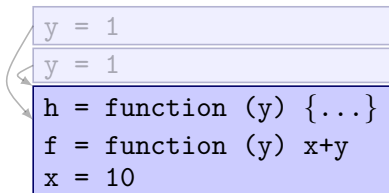
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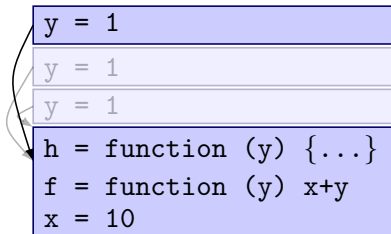
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What are the environments?



- ▶ `x ← 1`
- ▶ `f ← function (y) x + y`
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 x ← 10
 x + y
}`
- ▶ `f(1) ↦ 2`
- ▶ `h(1) ↦ 11`
- ▶ `f(1) ↦ 11`



What are the environments?



► $x \leftarrow 1$

What are the environments?



```
▶ x ← 1
▶ f ← function (y,recurse) {
    if (recurse) {
        x ← 10
        f(y,FALSE)
    } else {
        x+y
    }
}
```

What are the environments?



- ▶ $x \leftarrow 1$
- ▶ $f \leftarrow \text{function } (y, \text{recurse}) \{$
 - if (recurse) {
 - $x \leftarrow 10$
 - $f(y, \text{FALSE})$
 - } else {
 - $x+y$
 - }
- ▶ $f(2, \text{TRUE}) \mapsto 3$

What are the environments?



► $x \leftarrow 1$

What are the environments?



```
▶ x ← 1
▶ F ← function (x) {
    f ← function (y) {
        x+y
    }
    f
}
```

What are the environments?



- ▶ $x \leftarrow 1$
- ▶ $F \leftarrow \text{function } (x) \{$
 $f \leftarrow \text{function } (y) \{$
 $x+y$
 $\}$
 f
 $\}$
- ▶ $f \leftarrow F(2)$

What are the environments?



- ▶ $x \leftarrow 1$
- ▶ $F \leftarrow \text{function } (x) \{$
 $f \leftarrow \text{function } (y) \{$
 $x+y$
 $\}$
 f
 $\}$
- ▶ $f \leftarrow F(2)$
- ▶ $f(1) \mapsto 3$

What are the environments?



- ▶ $x \leftarrow 1$
- ▶ $F \leftarrow \text{function } (x) \{$
 $f \leftarrow \text{function } (y) \{$
 $x+y$
 $\}$
 f
 $\}$
- ▶ $f \leftarrow F(2)$
- ▶ $f(1) \mapsto 3$
- ▶ $x \leftarrow 10$

What are the environments?



- ▶ $x \leftarrow 1$
- ▶ $F \leftarrow \text{function } (x) \{$
 $f \leftarrow \text{function } (y) \{$
 $x+y$
 $\}$
 f
 $\}$
- ▶ $f \leftarrow F(2)$
- ▶ $f(1) \mapsto 3$
- ▶ $x \leftarrow 10$
- ▶ $f(1) \mapsto 3$

What are the environments?



► $x \leftarrow 1$

What are the environments?



```
▶ x ← 1
▶ F ← function() {
  x ← 10
  f_ ← function(y) {
    x + y
  }
  f_
}
```

What are the environments?



- ▶ `x ← 1`
- ▶ `F ← function() {
 x ← 10
 f_ ← function(y) {
 x + y
 }
 f_
}`
- ▶ `f ← F()`

What are the environments?



- ▶ $x \leftarrow 1$
- ▶ $F \leftarrow \text{function}() \{$
 - $x \leftarrow 10$
 - $f_ \leftarrow \text{function}(y) \{$
 - $x + y$
 - $\}$
 - $f_$
 - $\}$
- ▶ $f \leftarrow F()$
- ▶ $f(1) \mapsto 11$

What are the environments?



- ▶ $x \leftarrow 1$
- ▶ $F \leftarrow \text{function}() \{$
 - $x \leftarrow 10$
 - $f_ \leftarrow \text{function}(y) \{$
 - $x + y$
 - $\}$
 - $f_$
 - $\}$
- ▶ $f \leftarrow F()$
- ▶ $f(1) \mapsto 11$
- ▶ $x \leftarrow 100$

What are the environments?



- ▶ $x \leftarrow 1$
- ▶ $F \leftarrow \text{function}() \{$
 - $x \leftarrow 10$
 - $f_ \leftarrow \text{function}(y) \{$
 - $x + y$
 - $\}$
 - $f_$
- $\}$
- ▶ $f \leftarrow F()$
- ▶ $f(1) \mapsto 11$
- ▶ $x \leftarrow 100$
- ▶ $f(1) \mapsto 11$

What are the environments?



► $x \leftarrow 1$

What are the environments?



```
▶ x ← 1
▶ F ← function() {
  x ← 10
  g ← function(y) {
    x + y
  }
  f ← function(z) {
    x ← z
  }
  list(getter=g,setter=f)
}
```

What are the environments?



- ▶ `x ← 1`
- ▶ `F ← function() {
 x ← 10
 g ← function(y) {
 x + y
 }
 f ← function(z) {
 x ← z
 }
 list(getter=g,setter=f)
}`
- ▶ `o = F()`

What are the environments?



- ▶ `x ← 1`
- ▶ `F ← function() {
 x ← 10
 g ← function(y) {
 x + y
 }
 f ← function(z) {
 x ← z
 }
 list(getter=g,setter=f)
}`
- ▶ `o = F()`
- ▶ `o$getter(1) ↦ 11`

What are the environments?



- ▶ `x ← 1`
- ▶ `F ← function() {
 x ← 10
 g ← function(y) {
 x + y
 }
 f ← function(z) {
 x ← z
 }
 list(getter=g,setter=f)
}`
- ▶ `o = F()`
- ▶ `o$getter(1) ↦ 11`
- ▶ `o$setter(100)`

What are the environments?



- ▶ `x ← 1`
- ▶ `F ← function() {
 x ← 10
 g ← function(y) {
 x + y
 }
 f ← function(z) {
 x ← z
 }
 list(getter=g,setter=f)
}`
- ▶ `o = F()`
- ▶ `o$getter(1) ↦ 11`
- ▶ `o$setter(100)`
- ▶ `o$getter(1) ↦ 101`

Pass by reference (lists)

Reference to the object is passed
Original object can be modified

- ▶ `def f(l):`
 `l[2] = 0`
- ▶ `l = [1,2,3]`
- ▶ `f(l)`
- ▶ `l` \mapsto `[1,2,0]`

Pass by value (atoms)

Value of the object is copied
Original object cannot be
modified

- ▶ `def f(i):`
 `i = i+1`
- ▶ `i = 2`
- ▶ `f(i)`
- ▶ `i` \mapsto `2`

Semantically, pass by value for all basic data structures

- ▶ `f ← function(l) l[2] ← 0`
- ▶ `l = list(1,2,3)`
- ▶ `f(l)`
- ▶ `l ↦ 1 2 3`

Pass by promise

Arguments are evaluated only and when needed

```
▶ f ← function (x) {print(as.character(x)); x+10}
```

```
▶ g ← function (y,z) {print(as.character(y)); z}
```

```
▶ g(100,f(1))
```

```
  ↓  
"100"  
"1"  
11
```