

Functional programming in R (R'18)

TD 2: Vectors and matrices

Exercise 1 Using `help` find out the use of functions `sqrt`, `log10`, `log`, `exp`. Compute

$$\log_{10} \left(\sqrt{2} * \log_2 \left(e^{\sqrt{5} * \log_{10}(\sqrt{7})} \right) \right)$$

Exercise 2 Give examples that show the differences between any pair of the following functions `round`, `trunc`, `ceiling`, `floor`, and `signif`.

Exercise 3 Run and explain the following instructions:

```
x ← 1:10          sqrt(z)
y ← x * 2         log(x, base=2) * y
z ← x + y        round(log(z + exp(x)))
```

Exercise 4 Run and explain the following instructions

```
x ← c(3,6,8)      y ← c(2,5,1)      b ← x > 5
x[2]              x-y              x[b]
x[c(1,3)]         x*y              x[y>1.5]
x[-3]             x/y              y[x==6]
```

Exercise 5 Use Wikipedia to find area surfaces and populations of 6 countries: Poland, France, USA, Australia, and two countries of your choice. Put those values in two vectors `area` and `population`, and name the positions of the vectors accordingly. Calculate the densities of the countries and select countries with density less than 100 human inhabitants per km².

Exercise 6 Using the function `seq` create a vector `v` of 10 evenly-spaced numbers between 0 and 5. **Hint:** `seq(0,5,0.5)` is not the correct answer as it produces a vector of length 11.

Exercise 7 Using the vector `c(1,2,3)` how to obtain the vector `[1; 1; 2; 2; 3; 3]` by using only the function `rep`? **Hint:** check out the manual page of `rep`.

Exercise 8 Explain why the expression `1:10 * c(1,-1) + c(-1,0)` gives the vector `[0; -2; 2; -4; 4; -6; 6; -8; 8; -10]`. How to obtain the vector `[1; -1; 2; -2; 3; -3; 4; -4; 5; -5]` in a similar fashion?

Exercise 9 Using only the vectors `c(1,0)`, `c(0,1)`, and `1:10` together with the natural numbers and the standard operators `+`, `-`, `*`, and `/` construct the following vectors

- `[5; 15; 25; 35; 45; 55; 65; 75; 85; 95]`
- `[0.15; 0.25; 0.35; 0.45; 0.55; 0.65; 0.75; 0.85; 0.95; 1.05]`
- `[0; 1; 2; 3; 4; 5; 6; 7; 8; 9]`
- `[1; 0; 3; 0; 5; 0; 7; 0; 9; 0]`
- `[0; 2; 0; 4; 0; 6; 0; 8; 0; 10]`
- `[1; -2; 3; -4; 5; -6; 7; -8; 9; -10]`
- `[-1; 2; -3; 4; -5; 6; -7; 8; -9; 10]`
- `[1; 2; 0; 4; 5; 0; 7; 8; 0; 10]`
- `[0; 0; 0; 0; 0; 0; 0; 0; 0; 0]`
- `[1; 1; 1; 1; 1; 1; 1; 1; 1; 1]`

Exercise 10 Find what is the function `abs`. Why `y[y<0] ← -y[y<0]` has the same effect as `y ← abs(y)`?

Exercise 11 Using only function `paste`, the vector `c("x","y")`, and the regular sequence `1:5`, construct the vector

```
["x1";"y2";"x3";"y4";"x5"].
```

Using additionally the function `rep` how to obtain the vectors

```
["x1";"y2";"x3";"y4";"x5";"x1";"y2";"x3";"y4";"x5"]
```

and

```
["x1";"y1";"x2";"y2";"x3";"y3";"x4";"y4";"x5";"y5"].
```

Exercise 12 Run and explain the following instructions

```
x ← c(3,7,8,11)          sort(z)          sum(z) / sum(z)
y ← c(1,3)              sum(z)           mean(z)
z ← x + y               length(z)        mean(x) + mean(y)
```

Why are the two last results equal?

Exercise 13 Run and explain the following instructions

```
IQ ← c(71,109,113,105,109,79,95,78,88,142)  var(IQ)
mean(IQ)                                     sd(IQ)^2
sum(IQ)/length(IQ)                          sum((IQ - mean(IQ))^2) / (length(IQ) - 1)
median(IQ)                                   quantile(IQ,seq(0,1,0.1))
sd(IQ)                                       summary(IQ)
```

Exercise 14 Convert the vector `v` from the previous exercise to type `character` and assign the result to `w`. Then convert the vector `w` back to the type `numeric` and assign the result to `u`. Compare point-wise the values of `v` and `u` and explain the results. **Hint**: Check out the functions `as.type` for explicit type conversions and the operator `==` for point-wise comparison.

Exercise 15 We know that for $x \neq y$ the following two functions

$$f_1(x, y) = \frac{x^2 - y^2}{x - y} \quad \text{and} \quad f_2(x, y) = x + y$$

are identical. Compare the values (using the `==` operator) for $x = 1$ and $y = 1.1$ as well as for $x = 1$ and $y = 1.08$. Can you explain the results? **Hint**: It might help to change the number of digits displayed with `options(digits=n)`

Exercise 16 What is the smallest value of the function $f(x, y) = f_1(x, y) - f_2(x, y)$ that you can get? (when f has a direct implementation in R). **Hint**: look up the IEEE Standard for Floating-Point Arithmetic (IEEE 754), double precision format a.k.a. binary64.

Exercise 17 Run and explain the following instructions

```

matrix(c(0,1),3,4)          t(matrix(1:10,5,2))
matrix(c(0,1),2,4)        diag(4)
matrix(1:10,2,5)          matrix(1,2,2) %x% matrix(1:4,2,2)
matrix(1:10,2,5, byrow=TRUE) as.vector(t(matrix(1:4,2,2)))

```

Exercise 18 Research the function `outer` (or simply the operator `%o%` and use it to construct the multiplication table 10×10 .

Exercise 19 Use the following formula for maximum heart rate of an individual (due to Robergs and Landwehr)

$$HR_{\max} = 205.8 - (0.685 * age)$$

to construct a table (matrix) with goal heart rates for 1) warm-up (50% of HR), 2) weight control (60% HR), 3) aerobic (cardio) training (70% HR), and 4) anaerobic (lactic acid) training (80% HR), for individuals in ages between 20 and 70 (with resolution of 5 years). Compare it with the Haskell and Fox formula $HR_{\max} = 220 - age$.

Exercise 20 Imagine that we are purchasing a number of desktop computers for a company that consists of several departments. Each department has different needs and there are various configurations possible based on the components used. There are several distributors of components that sell them at different prices. Our task is identify a distributor from with whom we should make a batch order of all components and spend the least. A computer configuration is specified by the type of processor (standard and super-charged), the amount of RAM (multiples of 8GB), the size of the hard drive (250GB, 500GB, and 1TB), and whether or not it uses a discrete graphic card (on top of the integrated that comes with the processor). There are 3 configurations we wish to consider:

- The basic *office* configuration with standard processor, 8GB RAM, 250GB HDD, and no discrete graphic card. Perfect for internet browsing.
- The *power-house* configuration with super-charged processor, 16GB of RAM, 500GB HDD, and no discrete graphic card. Perfect for running long spreadsheet calculations.
- The *deluxe* configuration with super-charged processor, 32GB of RAM, 1TB HDD, and a discrete graphic cart. Perfect for working with graphics.

We have 4 departments: *Marketing*, *Accounting*, *Management*, and *Data Entry*. Each department requires different numbers of different configurations of computers:

- *Marketing* requires 3 office, 5 power-house, and 4 deluxe computers,
- *Accounting* requires 3 office, 20 power-house, and 1 deluxe computers,
- *Management* requires 2 office and 10 deluxe computers,
- *Data Entry* requires 20 office and 2 power-house computers.

We have 2 distributors *Materiel.net* and *GrosBill* and their prices are as follows:

	Standard CPU	Super CPU	8GB RAM	250GB HDD	500GB HDD	1TB HDD	GPU
Materiel.net	99.99	159.99	24.99	39.99	49.99	69.99	49.99
GrosBill	89.99	172.25	25.25	35.99	52.00	65.99	39.99

Use (vectors and) matrices to model the task at hand. Do not forget to give appropriate names to the columns and rows. Also, find how much of savings we could make if we were at liberty to order components from both distributors.