Functional programming in R (R'18)

TD 1: Higher-order functions and their types

In the following exercises we assume the following built-in functions with the corresponding types:

$\texttt{sum} : \texttt{num}^* \to \texttt{num}$	$\texttt{mean} : \texttt{num}^* \to \texttt{num}$	$\texttt{length} : \alpha^* \to \texttt{int}$
$\texttt{any} : \texttt{log}^* \to \texttt{log}$	$\texttt{all} : \texttt{log}^* \to \texttt{log}$	
$\texttt{substr} : \texttt{chr} \times \texttt{int} \times \texttt{int} \to \texttt{chr}$	$\texttt{paste} : \texttt{chr} \times \texttt{chr} \to \texttt{chr}$	
$* : \texttt{num} \times \texttt{num} \to \texttt{num}$	$+ : \texttt{num} \times \texttt{num} \to \texttt{num}$	$\texttt{ceil} : \texttt{num} \to \texttt{int}$

Additionally, we assume that the both operators $+$ and $*$ return an integer if both arguments are integers.

Exercise 1. Define the (most general) type of the following expressions

1. `function (x,y) ⟨x+y,x*y⟩`
2. `function (s,x,y) ⟨substr(s,x,y),y-x+1⟩`
3. `function (v) ⟨any(v),all(v)⟩`
4. `function (f,v) ⟨any(v),f(v)⟩`
5. `function (f,g,s) substr(s,f(s),g(s))`
6. `function (f,g,x) ⟨f(x),g(x)⟩`
7. `function (f,v) f(length(v)) + 1`
8. `function (f,v) length(f(v)) + 1`
9. `function (f,v) length(f(v+1))`
10. `function (v,f) ⟨sum(v),mean(v),f(v)⟩`
11. `function (f,v) ⟨sum(v),mean(v),f(v)⟩`
12. `function (f,g) function (v) ⟨mean(f(v)),all(g(v)),g(v),f(v)⟩`
13. `function (f,g,x) f(g(x)+1)`
14. `function (f,g,s) paste(f(s),g(s))`
15. `function (f,x,y,z) paste(paste(f(x+1),f(y+2)),f(z+3))`
16. `function (f,x,y,z) paste(paste(f(x),f(y)),f(z))`
17. `function (f,g,s) ⟨ceil(f(s)),paste(s,g(s))⟩`

Exercise 2. Propose the most specific type for the following functions:

1. `function (F,g,h,x) F(g(h(x)))`
2. `function (f,x) ⟨paste("Hello",f(x)),substr(f(x),0,3)⟩`
3. `function (f,g,x) ⟨f(x),g(x)⟩`
4. `function (F,g,x) (function (h) h(x))(F(g))`
5. `function (F,g,x) F(g)(x)`
6. `function (f,x,y,z) ⟨f(x),f(y),f(z)⟩`
7. `function (f,x,y,z) f(x,f(y,z))`
8. `function (F,g,x) ⟨F(g)(x),g(x)⟩`
9. `function (F,G,f,g,x) ⟨F(f)(x),G(f,g)(x),f(g(x))⟩`

Exercise 3. Define the (most general) type of the following expressions

1. `function (x) function (y) ⟨x+y,x*y⟩`
2. `function (s) function (x,y) ⟨substr(s,x,y),y-x+1⟩`
3. `function (v) ⟨any(v),all(v)⟩`
4. `function (f) function (v) ⟨any(v),f(v)⟩`
5. `(function (f) function (v) ⟨any(v),f(v)⟩)(all)`
6. `function (f) function (g) function (s) substr(s,f(s),g(s))`
7. `function (f) function (s,i,j) ⟨substr(s,f(i),f(j)),i+j⟩`
8. `function (f,g) function (x) ⟨f(x),g(x)⟩`
9. `function (f) function (v) f(length(v)) + 1`
10. `function (f) function (v) length(f(v)) + 1`
11. `function (f) function (v) length(f(v+1))`
12. `function (v) function (f) ⟨sum(v),mean(v),f(v)⟩`
13. `function (f) function (v) ⟨sum(v),mean(v),f(v)⟩`
14. `function (f) function (g) function (v) ⟨mean(f(v)),all(g(v)),g(v),f(v)⟩`
15. `function (f) function (g) function (x) f(g(x)+1)`
16. `function (f) function (g) function (s) paste(f(s),g(s))`
17. `function (f) function (x,y,z) paste(paste(f(x+1),f(y+2)),f(z+3))`
18. `function (f) function (x,y,z) paste(paste(f(x),f(y)),f(z))`
19. `function (f) function (g) function (s) ⟨ceil(f(s)),paste(s,g(s))⟩`
20. `function (f,g) function (x,y) ⟨mean(f(x)),g(x)+g(y)⟩`

Exercise 4. Propose the most specific type for the following functions:

1. `function (F) function (g) function (h) function (x) F(g(h(x)))`
2. `function (f) function (x) ⟨paste("Hello",f(x)),substr(f(x),0,3)⟩`
3. `function (f) function (g) function (x) ⟨f(x),g(x)⟩`
4. `function (F) function (g) function (x) (function (h) h(x))(F(g))`
5. `function (F) function (g) function (x) F(g)(x)`
6. `function (f) function (x,y,z) ⟨f(x),f(y),f(z)⟩`
7. `function (f) function (x,y,z) f(x,f(y,z))`
8. `function (F) function (g) function (x) ⟨F(g)(x),g(x)⟩`
9. `function (F,G) function (f,g) function (x) ⟨F(f)(x),G(f,g)(x),f(g(x))⟩`