



## R: Control and data flow

Sławek Staworko

Univ. Lille 3

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# Outline



Expressions

Function calls

Branching expression (if/else)

Loops

Scope and evaluation



# Expressions

## Functional programming paradigm

- ▶ Program is an expression  $\mathcal{E}$
- ▶ Running the program = evaluating the expression  $\mathcal{E}$

## Basic expression building blocks

- ▶ **Assignment**  $x \leftarrow \mathcal{E}$  assigns a value to the variable and evaluates to the value of  $x$
- ▶ **Function application**  $f(\mathcal{E}_1, \dots, \mathcal{E}_n)$  calls the function and evaluates to the result returned by the function
- ▶ **Composition**  $\mathcal{E}_0; \mathcal{E}_1$  evaluates to the value of the last expression  $\mathcal{E}_1$
- ▶ **Grouping**  $\{ \mathcal{E} \}$  evaluates to the inner expression (for structuring purposes)

## Example

►  $y \leftarrow x \leftarrow 1 \mapsto$

## Example

▶  $y \leftarrow x \leftarrow 1 \mapsto 1$   $x = 1$  and  $y = 1$

▶  $y \leftarrow 1 + (x \leftarrow 1) \mapsto$

## Example

- ▶  $y \leftarrow x \leftarrow 1 \mapsto 1 \quad x = 1 \text{ and } y = 1$
- ▶  $y \leftarrow 1 + (x \leftarrow 1) \mapsto 2 \quad x = 1 \text{ and } y = 2$
- ▶  $x \leftarrow 1; 0 \mapsto$

## Example

- ▶  $y \leftarrow x \leftarrow 1 \mapsto 1 \quad x = 1 \text{ and } y = 1$
- ▶  $y \leftarrow 1 + (x \leftarrow 1) \mapsto 2 \quad x = 1 \text{ and } y = 2$
- ▶  $x \leftarrow 1; 0 \mapsto 0 \quad x = 1$
- ▶  $y \leftarrow \{x \leftarrow 1; 0\} \mapsto$



## Example

- ▶  $y \leftarrow x \leftarrow 1 \mapsto 1 \quad x = 1 \text{ and } y = 1$
- ▶  $y \leftarrow 1 + (x \leftarrow 1) \mapsto 2 \quad x = 1 \text{ and } y = 2$
- ▶  $x \leftarrow 1; 0 \mapsto 0 \quad x = 1$
- ▶  $y \leftarrow \{x \leftarrow 1; 0\} \mapsto 0 \quad x = 1 \text{ and } y = 0$
- ▶  $y \leftarrow \{x \leftarrow \{1; 0\}\} \mapsto$

## Example

- ▶  $y \leftarrow x \leftarrow 1 \mapsto 1 \quad x = 1 \text{ and } y = 1$
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- ▶  $x \leftarrow 1; 0 \mapsto 0 \quad x = 1$
- ▶  $y \leftarrow \{x \leftarrow 1; 0\} \mapsto 0 \quad x = 1 \text{ and } y = 0$
- ▶  $y \leftarrow \{x \leftarrow \{1; 0\}\} \mapsto 0 \quad x = 0 \text{ and } y = 0$
- ▶  $2 + 3 \mapsto '+'(2,3) \mapsto 5$
- ▶  $\text{substr}(s \leftarrow "abcde", i \leftarrow \text{nchar}(s) - 2, i + 2) \mapsto$

## Example

- ▶  $y \leftarrow x \leftarrow 1 \mapsto 1$   $x = 1$  and  $y = 1$
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- ▶  $x \leftarrow 1; 0 \mapsto 0$   $x = 1$
- ▶  $y \leftarrow \{x \leftarrow 1; 0\} \mapsto 0$   $x = 1$  and  $y = 0$
- ▶  $y \leftarrow \{x \leftarrow \{1; 0\}\} \mapsto 0$   $x = 0$  and  $y = 0$
- ▶  $2 + 3 \mapsto '+'(2, 3) \mapsto 5$
- ▶  $\text{substr}(s \leftarrow \text{"abcde"}, i \leftarrow \text{nchar}(s) - 2, i + 2) \mapsto \text{"cde"}$   
 $s = \text{"abcde"}$  and  $i = 3$



# Function calls

# Positional and named arguments



- ▶ `f ← function (a,b) a + 3*b`
- ▶ `f(10,2) ↦ 16`
- ▶ `f(2,10) ↦ 32`
- ▶ `f(a=10,b=2) ↦ 16`
- ▶ `f(b=2,a=10) ↦ 16`

- ▶ `f ← function (a,b=1,c=3) {  
    if (b == 1) a + c else b + c  
}`
- ▶ `f(10) ↦ 13`
- ▶ `f(10,1) ↦ 13`
- ▶ `f(10,2) ↦ 5`
- ▶ `f(10,2,4) ↦ 6`
- ▶ `f(10,c=4) ↦ 14`
- ▶ `f(b=2,c=4) ↦ 6`
- ▶ `f(b=1) ↦ error`

## Ellipsis ...

Additional arguments of a function that can be passed on

## Example

```
sapply ← function(v,f,...) {  
  res ← vector()  
  for (i in 1:length(v)) {  
    res[i] ← f(v[i],...)  
  }  
  res  
}
```

```
sapply(1:4,function(x,y) x+y, 2) ↦ 3 4 5 6
```

```
sapply(1:4, '+', 2) ↦ 3 4 5 6
```



## Branching expression (if/else)



# Conditional expression (if/else)



if ( $\mathcal{E}$ )  $\mathcal{E}_1$  else  $\mathcal{E}_2$

- ▶ if  $\mathcal{E}$  evaluates to TRUE, evaluate and return the value of  $\mathcal{E}_1$ ; otherwise evaluate and return the value of  $\mathcal{E}_2$
- ▶  $\mathcal{E}$  must be interpretable as logical
- ▶ the else part is optional; when missing and the condition  $\mathcal{E}$  is not satisfied, the whole expression evaluates to NULL

## Example

- ▶ if (c(-1,1) > 0) "+" else "-"  $\mapsto$  "-"
- ▶ if (any(c(-1,1) > 0)) "+" else "-"  $\mapsto$  "+"
- ▶ if (0) "a" else "b"  $\mapsto$  "b"
- ▶ if (-2) "a" else "b"  $\mapsto$  "a"
- ▶ if ("FALSE") 1 else 0  $\mapsto$  0
- ▶ if ("a") 1 else 0  $\mapsto$  error

`ifelse(test, yes, no)`

- ▶ `test` an object which can be coerced to logical
- ▶ `yes` return values for true elements of `test`
- ▶ `no` return values for false elements of `test`

## Example

- ▶ `ifelse(-2:2 < 0, "-", "+")`  $\mapsto$  "-" "-" "+" "+" "+"
- ▶ `ifelse(1:4 %% 2 == 0, "E", "O")`  $\mapsto$  "O" "E" "O" "E"

# Vectorized if/else (contd.)



## Common misconception

```
ifelse(test, yes, no)
```

is supposed to be equivalent to

```
{tmp←yes; tmp[!test]←no; tmp}
```

## However, consider

- ▶ `x ← c(1,2,3,4,6,7,8,9)`
- ▶ `x %% 2 == 0`  $\mapsto$  F T F T T F T F
- ▶ `ifelse(x %% 2 == 0, c("E","e"), c("O","o","."))`  
 $\mapsto$  "O" "e" "." "e" "E" "." "E" "o"
- ▶ `{tmp←c("E","e"); tmp[x%%2!=0]←c("O","o","."); tmp}`  
 $\mapsto$  "O" "e" "o" NA NA "." NA "O"

# Switch statement



```
switch(x, "a", "b", "c")  
if (x == 1) {  
    "a"  
} else if (x == 2) {  
    "b"  
} else if (x == 3) {  
    "c"  
} else {  
    NULL  
}
```

```
switch(s, a=1, b=2, c=3, 4)  
if (s == "a") {  
    1  
} else if (s == "b") {  
    2  
} else if (s == "c") {  
    3  
} else {  
    4  
}
```



# Loops

## Two main looping mechanisms

- ▶ Imperative: `for`, `while`, and `repeat`
- ▶ Declarative: `apply` function family

## Kinds of loops

- ▶ `for (var in C) E` iterates over elements of a collection
- ▶ `while (cond) E` iterated as long as a given condition is satisfied
- ▶ `repeat E` iterates indefinitely (unless `break` is used)
- ▶ all loops evaluate to `NULL`

## Flow control inside loops

- ▶ `next` interrupts the current iteration and control flow moves to the next to next one
- ▶ `break` interrupts the execution and exits the inner most loop

# For loop



for ( $x$  in  $\mathcal{C}$ )  $\mathcal{E}$

- ▶  $\mathcal{C}$  is a vector or list (hence also factor and data frame)
- ▶  $\mathcal{E}$  is evaluated for  $x$  being assigned consecutive values in  $\mathcal{C}$
- ▶ side-effect: after having finished the execution the variable  $x$  is defined and carries the last assigned value



# For loop



`for (x in C) E`

- ▶  $C$  is a vector or list (hence also factor and data frame)
- ▶  $E$  is evaluated for  $x$  being assigned consecutive values in  $C$
- ▶ side-effect: after having finished the execution the variable  $x$  is defined and carries the last assigned value

## Example

```
sum ← function (v) {  
  acc ← 0;  
  for (x in v)  
    acc ← acc + x;  
  return(acc)  
}
```

`sum(c(1,4,2,6,1))`  $\mapsto$  14

# For loop



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## Example

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sum ← function (v) {  
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  for (x in v)  
    acc ← acc + x  
  acc  
}
```

sum(c(1,4,2,6,1))  $\mapsto$  14

# For loop (example)



## Example

```
f ← function (v) {  
  w ← numeric(length(v))  
  for (i in 1:length(v)) {  
    w[i] ← 2*v[i] + i  
  }  
  w  
}
```

$f(c(1,4,7)) \mapsto 3 \ 10 \ 17 \equiv 2*c(1,4,7) + 1:3$

# For loop (examples)



## Example

```
find_elem ← function (v,x) {  
  for (i in 1:length(v)) {  
    if (v[i] == x) {  
      return(TRUE)  
    }  
  }  
  FALSE  
}  
find_elem(c(1,4,7,10,3,2,1,4),2) ↦ TRUE  
find_elem(v,x) ≡ any(v == x)
```

# For loop (examples)



## Example

```
find_pos ← function (v,x) {  
  for (i in 1:length(v)) {  
    if (v[i] == x) {  
      return(i)  
    }  
  }  
}
```

```
find_pos(c(1,4,7,10,3,2,1,4),2) ↦ 6
```

```
find_pos(v,x) ≡ (1:length(v))[v == x][1]
```

# For loop (example)



## Example

```
sapply ← function(v,f) {  
  res ← vector()  
  for (i in 1:length(v)) {  
    res[i] ← f(v[i])  
  }  
  res  
}
```

`sapply(1:4,function (x) x^2)`  $\mapsto$  1 4 9 16

`sapply(1:4,as.character)`  $\mapsto$  "1" "2" "3" "4"

`sapply(1:4,function (x) x/2.0)`  $\mapsto$  0.5 1.0 1.5 2.0

# While loop



```
while (cond)  $\mathcal{E}$ 
```

- ▶ execute  $\mathcal{E}$  again and again as long as cond evaluates to TRUE

# While loop (example)



## Example

```
create_polynomial ← function (p) function (x) {  
  y ← 0  
  i ← length(p)  
  while (i > 0) {  
    y ← y + p[i] * (x^(length(p)-i))  
    i ← i - 1  
  }  
  y  
}
```

```
p ← create_polynomial(c(5,4,2,3))
```

$p(x) = 5x^3 + 4x^2 + 2x + 3$

$p(1) \mapsto 14$

$p(-1) \mapsto 0$

$p(2) \mapsto 63$



# Repeat loop



repeat  $\mathcal{E}$

- ▶ execute  $\mathcal{E}$  again and again until `break` is called

## Repeat loop (example)



### Example

```
find_root ← function(p, x1, x2) {  
  repeat {  
    y1 ← p(x1)  
    y2 ← p(x2)  
    xz ← (x1+x2)/2  
    yz ← p(xz)  
    if (sign(y1) == sign(yz))  
      x1 ← xz  
    if (sign(y2) == sign(yz))  
      x2 ← xz  
    if (abs(yz) < 10e-10)  
      break  
  }  
  xz  
}  
find_root(create_polynomial(c(5,4,2,3),-10,10)) ↦ -1
```



## Apply family of functions

- ▶ `sapply` operates on vectors
- ▶ `lapply` operates on list
- ▶ `apply` operates on matrices
- ▶ `mapply` operates on multiple vectors

## Ellipsis

Additional arguments that are passed through to other functions.

## Example

```
sapply ← function(v,f,...) {  
  res ← vector()  
  for (i in 1:length(v)) {  
    res[i] ← f(v[i],...)  
  }  
  res  
}
```

```
sapply(1:4,function(x,y) x+y, 2) ↦ 3 4 5 6  
sapply(1:4,'+',2) ↦ 3 4 5 6
```

# Use case: function vectorisation

## Most functions in R are vectorised

- ▶ when given a vector, the function is applied on every element
- ▶ `sqrt` :  $\text{num}^* \rightarrow \text{num}^*$
- ▶ `sqrt(c(1,2,3))`  $\mapsto$  1.000000 1.414214 1.732051

## Non-vectorised function can be vectorised with `sapply`

- ▶ 

```
loop ← function (n) {  
  x ← 1  
  for (i in 1:n)  
    x ← sin(x)  
  x  
}
```
- ▶ `loop(1)`  $\mapsto$  0.8414710
- ▶ `loop(2)`  $\mapsto$  0.7456241
- ▶ `loop(c(1,2))`  $\mapsto$  **error**
- ▶ `loop_vect ← function(v) sapply(v,loop)`
- ▶ `loop_vect(c(1,2))`  $\mapsto$  0.8414710 0.7456241

`sapply(v,f)` will return a matrix

- ▶ when `f` returns a vector of the same length each time

## Example

- ▶ `f ← function (n) cos(seq(n, (n+1), 0.5))`
- ▶ `f(1) ↦ 0.54030 0.07074 -0.41615`
- ▶ `f(2) ↦ -0.4161 -0.8011 -0.9900`
- ▶ `sapply(1:4,f)`

$$\begin{array}{c} \Downarrow \\ \left[ \begin{array}{cccc} 0.54030 & -0.4161 & -0.9900 & -0.6536 \\ 0.07074 & -0.8011 & -0.9365 & -0.2108 \\ -0.41615 & -0.9900 & -0.6536 & 0.2837 \end{array} \right] \end{array}$$

`apply(M,dim,f,...)`

- ▶ if `dim = 1`, then `f` is called on every row
- ▶ if `dim = 2`, then `f` is called on every column
- ▶ if `dim = c(1,2)`, then `f` is called on every cell

## Example

- ▶ `m ←`  $\begin{bmatrix} 1 & 4 & 9 \\ 4 & 16 & 25 \end{bmatrix}$
- ▶ `apply(m,1,sum) ↦` 14 45
- ▶ `apply(m,2,sum) ↦` 5 20 34
- ▶ `apply(m,c(1,2),sqrt) ↦`  $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}$

```
mapply(f, v1, v2, ..., vk, ...)
```

- ▶ constructs a vector  $v$  such that
- ▶  $v[i] \leftarrow f(v_1[i], v_2[i], \dots, v_k[i], \dots)$
- ▶  $v$  is as long as the longest of the input vectors (recycling is applied if lengths are not all equal)

## Example

- ▶ `mapply(function (x,y) x+y, c(1,3), c(4,6))`  $\mapsto$  5 9  
 $\equiv$  `mapply('+', c(1,3), c(4,6))`  $\equiv$  `c(1,3) + c(4,6)`
- ▶ 

```
f ← function (x,y) {  
  for (i in 1:y)  
    x ← sqrt(x+1)  
  x  
}
```
- ▶ `mapply(f, 1:3, c(1,5,10))`  $\mapsto$  1.4142 1.6191 1.6180



# Scope and evaluation

## Scoping

- ▶ what a "*part of a program*" means?
- ▶ variable: where is its value stored

## Two types of scoping

**lexical** depends on the location in the source code, where the variable is defined

**dynamic** depends on the execution context

- ▶ *environment* is a frame of reference for variable lookup
- ▶ organized into a *stack* (but actually a *tree*)
- ▶ if a variable doesn't exist in the current environment, then check its parent, then its grandparent, etc.
- ▶ *global* environment, storing all global variables, is at the end of the search path (*root* environment)
- ▶ when function is called a *new environment* is created, function parameters are new variables in the new environment
- ▶ the parent environment of a function call is always the environment where the function has been *defined* (and **not** where the function has been called)

# What are the environments?



►  $x \leftarrow 1$

$x = 1$

# What are the environments?



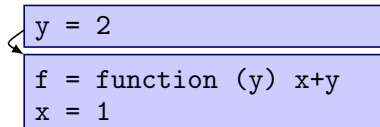
- ▶ `x ← 1`
- ▶ `f ← function (y) x + y`

```
f = function (y) x+y  
x = 1
```

# What are the environments?



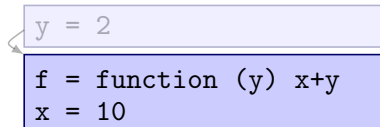
- ▶ `x ← 1`
- ▶ `f ← function (y) x + y`
- ▶ `f(2) ↦ 3`



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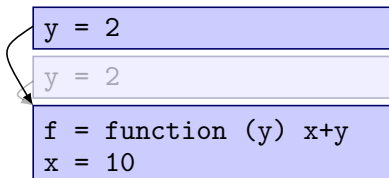
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- ▶ `f(2) ↦ 3`
- ▶ `x ← 10`



# What are the environments?



- ▶ `x ← 1`
- ▶ `f ← function (y) x + y`
- ▶ `f(2) ↦ 3`
  
- ▶ `x ← 10`
- ▶ `f(2) ↦ 12`





# What are the environments?



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# What are the environments?



- ▶ `x ← 1`
- ▶ `f ← function (y) x + y`

```
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```

# What are the environments?



```
▶ x ← 1
▶ f ← function (y) x + y
▶ g ← function (y) {
  x ← 10
  x + y
}
```

```
g = function (y) {...}
f = function (y) x+y
x = 1
```

# What are the environments?



- ▶ `x ← 1`
- ▶ `f ← function (y) x + y`
- ▶ `g ← function (y) {`  
    `x ← 10`  
    `x + y`  
}
- ▶ `f(1) ↦ 2`

```
y = 1
g = function (y) {...}
f = function (y) x+y
x = 1
```

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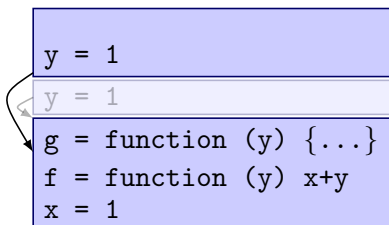
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    x + y  
}`
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```

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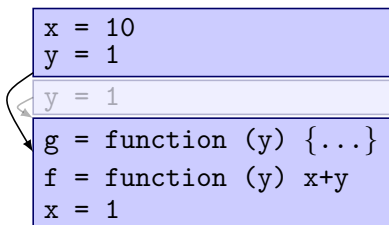
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}
- ▶ `f(1) ↦ 2`
- ▶ `g(1) ↦ 11`



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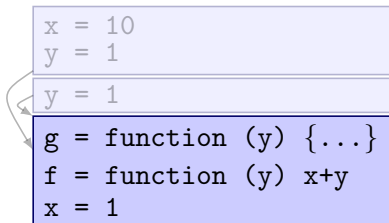
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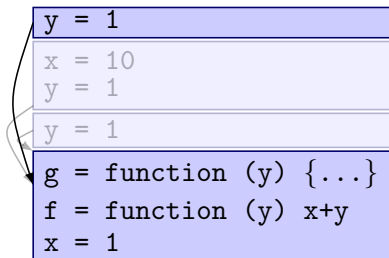




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# What are the environments?



```
▶ x ← 1
▶ f ← function (y) x + y
▶ h ← function (y) {
  x ← 10
  x + y
}
```

```
h = function (y) {...}
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x = 1
```

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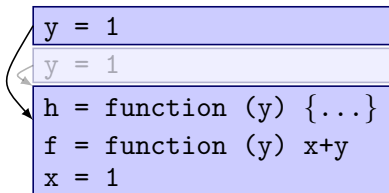
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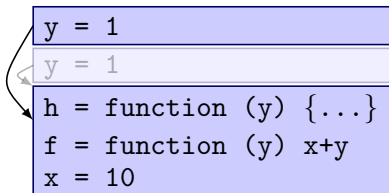
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    x + y  
}`
- ▶ `f(1) ↦ 2`
- ▶ `h(1) ↦ 11`

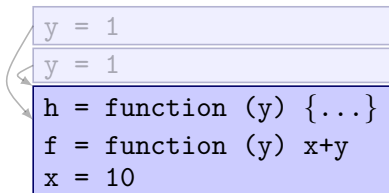




# What are the environments?



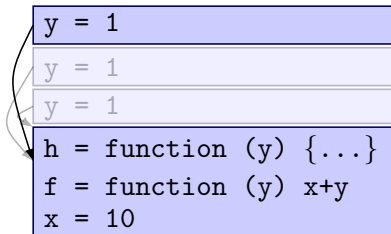
- ▶ `x ← 1`
- ▶ `f ← function (y) x + y`
- ▶ `h ← function (y) {  
    x ← 10  
    x + y  
}`
- ▶ `f(1) ↦ 2`
- ▶ `h(1) ↦ 11`



# What are the environments?



- ▶ `x ← 1`
- ▶ `f ← function (y) x + y`
- ▶ `h ← function (y) {  
    x ← 10  
    x + y  
}`
- ▶ `f(1) ↦ 2`
- ▶ `h(1) ↦ 11`
- ▶ `f(1) ↦ 11`



# What are the environments?



►  $x \leftarrow 1$

# What are the environments?



```
▶ x ← 1
▶ f ← function (y,recuse) {
    if (recuse) {
        x ← 10
        f(y,FALSE)
    } else {
        x+y
    }
}
```

# What are the environments?



- ▶  $x \leftarrow 1$
- ▶  $f \leftarrow \text{function } (y, \text{recurse}) \{$ 
  - if (recurse) {
  - $x \leftarrow 10$
  - $f(y, \text{FALSE})$
  - } else {
  - $x+y$
  - }
  - }
- ▶  $f(2, \text{TRUE}) \mapsto 3$

# What are the environments?



►  $x \leftarrow 1$

# What are the environments?



```
▶ x ← 1
▶ F ← function (x) {
    f ← function (y) {
        x+y
    }
    f
}
```

# What are the environments?



- ▶  $x \leftarrow 1$
- ▶  $F \leftarrow \text{function } (x) \{$   
     $f \leftarrow \text{function } (y) \{$   
         $x+y$   
     $\}$   
     $f$   
   $\}$
- ▶  $f \leftarrow F(2)$



# What are the environments?



- ▶  $x \leftarrow 1$
- ▶  $F \leftarrow \text{function } (x) \{$   
     $f \leftarrow \text{function } (y) \{$   
         $x+y$   
     $\}$   
     $f$   
     $\}$
- ▶  $f \leftarrow F(2)$
- ▶  $f(1) \mapsto 3$

# What are the environments?



- ▶  $x \leftarrow 1$
- ▶  $F \leftarrow \text{function } (x) \{$   
     $f \leftarrow \text{function } (y) \{$   
         $x+y$   
     $\}$   
     $f$   
     $\}$
- ▶  $f \leftarrow F(2)$
- ▶  $f(1) \mapsto 3$
- ▶  $x \leftarrow 10$

# What are the environments?



- ▶  $x \leftarrow 1$
- ▶  $F \leftarrow \text{function } (x) \{$   
     $f \leftarrow \text{function } (y) \{$   
         $x+y$   
     $\}$   
     $f$   
     $\}$
- ▶  $f \leftarrow F(2)$
- ▶  $f(1) \mapsto 3$
- ▶  $x \leftarrow 10$
- ▶  $f(1) \mapsto 3$

# What are the environments?



►  $x \leftarrow 1$

# What are the environments?



```
▶ x ← 1
▶ F ← function() {
  x ← 10
  f_ ← function(y) {
    x + y
  }
  f_
}
```

# What are the environments?



- ▶ `x ← 1`
- ▶ `F ← function() {  
 x ← 10  
 f_ ← function(y) {  
 x + y  
 }  
 f_  
}`
- ▶ `f ← F()`

# What are the environments?



- ▶  $x \leftarrow 1$
- ▶  $F \leftarrow \text{function}() \{$   
     $x \leftarrow 10$   
     $f_ \leftarrow \text{function}(y) \{$   
         $x + y$   
     $\}$   
     $f_$   
   $\}$
- ▶  $f \leftarrow F()$
- ▶  $f(1) \mapsto 11$

# What are the environments?



- ▶ `x ← 1`
- ▶ `F ← function() {  
    x ← 10  
    f_ ← function(y) {  
        x + y  
    }  
    f_  
}`
- ▶ `f ← F()`
- ▶ `f(1) ↦ 11`
- ▶ `x ← 100`



# What are the environments?



- ▶ `x ← 1`
- ▶ `F ← function() {  
 x ← 10  
 f_ ← function(y) {  
 x + y  
 }  
 f_  
}`
- ▶ `f ← F()`
- ▶ `f(1) ↦ 11`
- ▶ `x ← 100`
- ▶ `f(1) ↦ 11`

# What are the environments?



►  $x \leftarrow 1$

# What are the environments?



```
▶ x ← 1
▶ F ← function() {
  x ← 10
  g ← function(y) {
    x + y
  }
  f ← function(z) {
    x ← z
  }
  list(getter=g,setter=f)
}
```

# What are the environments?



- ▶ `x ← 1`
- ▶ `F ← function() {  
 x ← 10  
 g ← function(y) {  
 x + y  
 }  
 f ← function(z) {  
 x ← z  
 }  
 list(getter=g,setter=f)  
}`
- ▶ `o = F()`

# What are the environments?



- ▶ `x ← 1`
- ▶ `F ← function() {  
 x ← 10  
 g ← function(y) {  
 x + y  
 }  
 f ← function(z) {  
 x ← z  
 }  
 list(getter=g,setter=f)  
}`
- ▶ `o = F()`
- ▶ `o$getter(1) ↦ 11`

# What are the environments?



- ▶ `x ← 1`
- ▶ `F ← function() {  
 x ← 10  
 g ← function(y) {  
 x + y  
 }  
 f ← function(z) {  
 x ← z  
 }  
 list(getter=g,setter=f)  
}`
- ▶ `o = F()`
- ▶ `o$getter(1) ↦ 11`
- ▶ `o$setter(100)`

# What are the environments?



- ▶ `x ← 1`
- ▶ `F ← function() {  
 x ← 10  
 g ← function(y) {  
 x + y  
 }  
 f ← function(z) {  
 x ← z  
 }  
 list(getter=g,setter=f)  
}`
- ▶ `o = F()`
- ▶ `o$getter(1) ↦ 11`
- ▶ `o$setter(100)`
- ▶ `o$getter(1) ↦ 101`

## Pass by reference (lists)

Reference to the object is passed  
Original object can be modified

- ▶ `def f(l):`  
    `l[2] = 0`
- ▶ `l = [1,2,3]`
- ▶ `f(l)`
- ▶ `l`  $\mapsto$  `[1,2,0]`

## Pass by value (atoms)

Value of the object is copied  
Original object cannot be  
modified

- ▶ `def f(i):`  
    `i = i+1`
- ▶ `i = 2`
- ▶ `f(i)`
- ▶ `i`  $\mapsto$  `2`



Semantically, pass by value for all basic data structures

- ▶ `f ← function(l) l[2] ← 0`
- ▶ `l = list(1,2,3)`
- ▶ `f(l)`
- ▶ `l ↦ 1 2 3`

## Pass by promise

Arguments are evaluated only and when needed

```
▶ f ← function (x) {print(as.character(x)); x+10}
```

```
▶ g ← function (y,z) {print(as.character(y)); z}
```

```
▶ g(100,f(1))
```

```
  ↓  
"100"  
"1"  
11
```