

Functional programming in R (R'18)

TD 1: Higher-order functions and their types

In the following exercises we assume the following built-in function with the following types:

$\text{sum} : \text{num}^* \rightarrow \text{num}$	$\text{mean} : \text{num}^* \rightarrow \text{num}$	$\text{length} : \alpha^* \rightarrow \text{int}$
$\text{any} : \text{log}^* \rightarrow \text{log}$	$\text{all} : \text{log}^* \rightarrow \text{log}$	
$\text{substr} : \text{chr} \times \text{int} \times \text{int} \rightarrow \text{chr}$	$\text{paste} : \text{chr} \times \text{chr} \rightarrow \text{chr}$	
$* : \text{num} \times \text{num} \rightarrow \text{num}$	$+ : \text{num} \times \text{num} \rightarrow \text{num}$	$\text{ceil} : \text{num} \rightarrow \text{int}$

We assume that the operators $+$ and $*$ return an integer if both arguments are integers.

Define the (most general) type of the following functions

1. $\text{function } (x,y) \langle x+y,x*y \rangle$
2. $\text{function } (x) \text{ function } (y) \langle x+y,x*y \rangle$
3. $\text{function } (s) \text{ function } (x,y) \langle \text{substr}(s,x,y),y-x+1 \rangle$
4. $\text{function } (v) \langle \text{any}(v),\text{all}(v) \rangle$
5. $\text{function } (f) \text{ function } (v) \langle \text{any}(v),f(v) \rangle$
6. $(\text{function } (f) \text{ function } (v) \langle \text{any}(v),f(v) \rangle)(\text{all})$
7. $\text{function } (f) \text{ function } (g) \text{ function } (s) \text{ substr}(s,f(s),g(s))$
8. $\text{function } (f) \text{ function } (s,i,j) \langle \text{substr}(s,f(i),f(j)),i+j \rangle$
9. $\text{function } (f,g) \text{ function } (x) \langle f(x),g(x) \rangle$
10. $\text{function } (f) \text{ function } (v) f(\text{length}(v)) + 1$
11. $\text{function } (f) \text{ function } (v) \text{length}(f(v)) + 1$
12. $\text{function } (f) \text{ function } (v) \text{length}(f(v+1))$
13. $\text{function } (v) \text{ function } (f) \langle \text{sum}(v),\text{mean}(v),f(v) \rangle$
14. $\text{function } (f) \text{ function } (v) \langle \text{sum}(v),\text{mean}(v),f(v) \rangle$
15. $\text{function } (f) \text{ function } (g) \text{ function } (v) \langle \text{mean}(f(v)),\text{all}(g(v)),g(v),f(v) \rangle$
16. $\text{function } (f) \text{ function } (g) \text{ function } (x) f(g(x)+1)$
17. $\text{function } (f) \text{ function } (g) \text{ function } (s) \text{ paste}(f(s),g(s))$
18. $\text{function } (f) \text{ function } (x,y,z) \text{ paste}(\text{paste}(f(x+1),f(y+2)),f(z+3))$
19. $\text{function } (f) \text{ function } (x,y,z) \text{ paste}(\text{paste}(f(x),f(y)),f(z))$
20. $\text{function } (f) \text{ function } (g) \text{ function } (s) \langle \text{ceil}(f(s)),\text{paste}(s,g(s)) \rangle$
21. $\text{function } (f,g) \text{ function } (x,y) \langle \text{mean}(f(x)),g(x)+g(y) \rangle$
22. $\text{function } (F) \text{ function } (g) \text{ function } (h) \text{ function } (x) F(g(h(x)))$
23. $\text{function } (f) \text{ function } (x) \langle \text{paste}(\text{"Hello"},f(x)),\text{substr}(f(x),0,3) \rangle$
24. $\text{function } (f) \text{ function } (g) \text{ function } (x) \langle f(x),g(x) \rangle$
25. $\text{function } (F) \text{ function } (g) \text{ function } (x) (\text{function } (h) h(x))(F(g))$
26. $\text{function } (F) \text{ function } (g) \text{ function } (x) F(g)(x)$
27. $\text{function } (f) \text{ function } (x,y,z) \langle f(x),f(y),f(z) \rangle$
28. $\text{function } (f) \text{ function } (x,y,z) f(x,f(y,z))$
29. $\text{function } (F) \text{ function } (g) \text{ function } (x) \langle F(g)(x),g(x) \rangle$
30. $\text{function } (F,G) \text{ function } (f,g) \text{ function } (x) \langle F(f)(x),G(f,g)(x),f(g(x)) \rangle$