New Control Schemes for Bilateral Teleoperation under Asymmetric Communication Channels: Stabilization and Performance under Variable Time Delays

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PLAN

1. PRELIMINARIES
2. NOVEL CONTROL SCHEMES
3. ROBUSTNESS ASPECTS
4. EXPERIMENTATIONS
5. CONCLUSIONS & PERSPECTIVES
1. Preliminaries

😊 Background & Challenges;
😊 Delayed Teleoperation;
😊 Positioning;
😊 Modeling of Time Delays;
😊 Stability of Time Delay Systems;
Cooperative System :

Haptic interface cooperative system, LAGIS

Performance Objectives :

😊 Stability;
😊 Synchronization (the position tracking, from the master to the slave);
😊 Transparency (the force tracking, from the slave to the master);
Cooperative System:

Haptic interface cooperative system, LAGIS

Background & Challenges:
- Wireless or long-distance;
- Complex environment;
- Modeling uncertainties;
- Collaborative sensing;
- Path planning and trajectory tracking;
1. Preliminaries

- Background & Challenges;
- Delayed Teleoperation;
- Positioning;
- Modeling of Time Delays;
- Stability of Time Delay Systems;
Delayed Teleoperation - Two Cases:

Unilateral master/slave teleoperation

Bilateral master/slave teleoperation
Delayed Teleoperation - Two Cases:

Unilateral master/slave teleoperation

Bilateral master/slave teleoperation
Bilateral Teleoperation:

1. **Velocity**
   - Human Operator $ightarrow$ Master + Controller $ightarrow$ Network $ightarrow$ Slave + Controller $ightarrow$ Env.
   - Velocity flows in the same direction as force.

2. **Force**
   - Human Operator $ightarrow$ Master + Controller $ightarrow$ Network $ightarrow$ Slave + Controller $ightarrow$ Env.
   - Force flows in the opposite direction of velocity.

3. **Position**
   - Human Operator $ightarrow$ Master + Controller $ightarrow$ Network $ightarrow$ Slave + Controller $ightarrow$ Env.
   - Position flows in the same direction as force.

The diagrams illustrate the flow of information and control in a bilateral teleoperation system under asymmetric communication channels.
Bilateral Teleoperation:

Position/Velocity

Humain Operator → Master + Controller → Network → Slave + Controller → Env.

Position/Velocity

Humain Operator → Master + Controller → Network → Slave + Controller → Env.

Position/Velocity/Force

Humain Operator → Master + Controller → Network → Slave + Controller → Env.

Force
Preliminaries

Delayed Teleoperation

General Teleoperation Structure:

- $F_h(t)$
- $x_m(t)$
- Master
- Master Controller
- Network
- $\tau_1(t)$
- $\tau_2(t)$
- $F_s(t)$
- $x_s(t)$
- Slave
- Slave Controller
- Env.
- $F_e(t)$
- $x_s(t)$

Symbols:
- $F$ $\longrightarrow$ force;
- $x$ $\longrightarrow$ position/velocity;
- $\tau$ $\longrightarrow$ delay;
- $m, s, h, e$ $\longrightarrow$ master, slave, human operator, environment;
General Teleoperation Structure:

- $F_h(t)$ and $x_m(t)$ from Human Operator to Master
- $F_m(t)$ from Master to Slave
- $F_s(t)$ and $x_s(t)$ from Slave to Environment
- $\tau_1(t)$ and $\tau_2(t)$ from Network

Symbols:
- $F$ → force
- $x$ → position/velocity
- $\tau$ → delay
- $m$, $s$, $h$, $e$ → master, slave, human operator, environment
New Control Schemes for Bilateral Teleoperation under Asymmetric Communication Channels

Preliminaries

Delayed Teleoperation

General Teleoperation Structure:

-微笑 $F_h(t)$
-微笑 $x_m(t)$
-微笑 $F_m(t)$
-微笑 $x_m(t)$
-微笑 $F_s(t)$
-微笑 $x_s(t)$
-微笑 $F_e(t)$

-微笑 $F \longrightarrow$ force;
-微笑 $x \longrightarrow$ position/velocity;
-微笑 $\tau \longrightarrow$ delay;
-微笑 $m, s, h, e \longrightarrow$ master, slave, human operator, environment;
Preliminaries

Delayed Teleoperation

General Teleoperation Structure:

- $F$ $\rightarrow$ force;
- $x$ $\rightarrow$ position/velocity;
- $\tau$ $\rightarrow$ delay;
- $m$, $s$, $h$, $e$ $\rightarrow$ master, slave, human operator, environment;
New Control Schemes for Bilateral Teleoperation under Asymmetric Communication Channels

**Preliminaries**

*Delayed Teleoperation*

**General Teleoperation Structure:**

- $F_h(t)$: Force from human operator
- $x_m(t)$: Position/velocity from master
- $x_m(t)$: Position/velocity to master
- $F_m(t)$: Force to master

- $	au_1(t)$: First network delay
- $	au_2(t)$: Second network delay

- $F_s(t)$: Force from slave
- $x_s(t)$: Position/velocity from slave
- $x_s(t)$: Position/velocity to slave
- $F_e(t)$: Environment force

- $m$, $s$, $h$, $e$: Master, slave, human operator, environment

- $F$: Force
- $x$: Position/velocity
- $\tau$: Delay

Symbols:
- $\smiley F \rightarrow$ force;
- $\smiley x \rightarrow$ position/velocity;
- $\smiley \tau \rightarrow$ delay;
- $\smiley m, s, h, e \rightarrow$ master, slave, human operator, environment;
**General Teleoperation Structure:**

- **$F_h(t)$** and **$x_m(t)$** as inputs to the **Master Controller**
- **$F_m(t)$** and **$x_m(t)$** as outputs from the **Master Controller**
- **$F_s(t)$** and **$x_s(t)$** as inputs to the **Slave Controller**
- **$F_e(t)$** and **$x_s(t)$** as outputs from the **Slave Controller**

**Performance Objectives:**

- **Stability**;
- Synchronization (the position tracking, from the master to the slave);
- Transparency (the force tracking, from the slave to the master);

**Properties:**

1. Linear master/slave systems;
2. Internet/Ethernet/Wifi…;
3. Time-stamped data packets;
1. Preliminaries

- Background & Challenges;
- Delayed Teleoperation;
- **Positioning**;
- Modeling of Time Delays;
- Stability of Time Delay Systems;
Main Capabilities of Recent Control Strategies:

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<thead>
<tr>
<th>Control Strategy</th>
<th>Time Delays</th>
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S. Zampieri, IFAC, 2008;
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Goals of this thesis: ✓ ✓ ✓ ✓ + Performance

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1. Preliminaries

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- Stability of Time Delay Systems;
Network Induced Delays:

Communication delay $\tau^c(t)$

Asynchronous sampling delay $\tau^s(t)$

Data loss delay
Network Induced Delays:

\[ \tau(t) = \tau^c(t) + \tau^s(t) + NT. \]  

Notations and Constraints for Delays:

\[ \tau(t) = h + \eta(t). \]  

- Constant delay: \( \eta(t) = 0, h \neq 0 \);
- Non-small time-varying delay:
  \( |\eta(t)| \leq \mu < h \), so
  \( \tau(t) \in [h - \mu, h + \mu] \);
- Interval time-varying delay:
  \( 0 \leq \eta(t) \leq \mu \), so \( \tau(t) \in [h, h + \mu] \);
- Time delay with the constraint on the derivative:
  \( \dot{\tau}(t) \leq d < 1 \),
  \( d > 0 \), or \( \dot{\tau}(t) \leq 1 \);
Network Induced Delays:

\[ \tau(t) = \tau^c(t) + \tau^s(t) + NT. \quad (1) \]

Notations and Constraints for Delays:

\[ \tau(t) = h + \eta(t). \quad (2) \]

- Constant delay: \( \eta(t) = 0, h \neq 0 \);
- Non-small time-varying delay: 
  \[ |\eta(t)| \leq \mu < h, \text{ so } \tau(t) \in [h - \mu, h + \mu]; \]
- Interval time-varying delay:
  \[ 0 \leq \eta(t) \leq \mu, \text{ so } \tau(t) \in [h, h + \mu]; \]
- Time delay with the constraint on the derivative: 
  \[ \dot{\tau}(t) \leq d < 1, \quad d > 0, \text{ or } \dot{\tau}(t) \leq 1; \]
1. Preliminaries

- Background & Challenges;
- Delayed Teleoperation;
- Positioning;
- Modeling of Time Delays;
- Stability of Time Delay Systems;
New Control Schemes for Bilateral Teleoperation under Asymmetric Communication Channels

PRELIMINARIES

Stability of Time Delay Systems

Delay-Independent Stability:
If a time-delay system is asymptotically stable for any delay values belonging to $\mathbb{R}_+$, the system is said to be delay-independent asymptotically stable.

Delay-Dependent Stability:
If a time-delay system is asymptotically stable for all delay values belonging to a compact subset $D$ of $\mathbb{R}_+$, the system is said to be delay-dependent asymptotically stable.

Rate-Independent Stability:
For a delay-dependent asymptotically stable time delay system, if the stability does not depend on the variation rate of delays or on the time derivative of delays, the system is said to be rate-independent asymptotically stable.

Rate-Dependent Stability:
For a delay-dependent asymptotically stable time delay system, if the stability depends on the variation rate of delays or on the time derivative of delays, the system is said to be rate-dependent asymptotically stable.
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For a delay-dependent asymptotically stable time delay system, if the stability does not depend on the variation rate of delays or on the time derivative of delays, the system is said to be rate-independent asymptotically stable.

Rate-Dependent Stability:
For a delay-dependent asymptotically stable time delay system, if the stability depends on the variation rate of delays or on the time derivative of delays, the system is said to be rate-dependent asymptotically stable.
General System with Piecewise Time-Varying Delays:

\[
\begin{align*}
\dot{x}(t) &= f(x(t), x(t - \tau_1(t)), u(t), u(t - \tau_2(t))), \\
x(t_0 + \theta) &= \phi(\theta), \quad \dot{x}(t_0 + \theta) = \dot{\phi}(\theta), \\
u(t_0 + \theta) &= \zeta(\theta), \\
\theta &\in [-h, 0].
\end{align*}
\]  

(3)

Linear Case:

\[
\begin{align*}
\dot{x}(t) &= A_0 x(t) + \sum_{i=1}^{n} A_i x(t - \tau_{1i}(t)) + B_0 u(t) + \sum_{j=1}^{m} B_j u(t - \tau_{2j}(t)), \\
x(t_0 + \theta) &= \phi(\theta), \quad \dot{x}(t_0 + \theta) = \dot{\phi}(\theta), \quad \theta \in [-h, 0].
\end{align*}
\]  

(4)
Lyapunov-Krasovskii Functionals (LKF) - Linear Case:

Search of suitable $V(x(t), \dot{x}(t))$ [E. Fridman, IMA Journal of Mathematical Control and Information, 2006] : 

$$
V(x(t), \dot{x}(t)) = x(t)^T P x(t) + \int_{t-h_2}^{t} x(s)^T S_a x(s) ds + \int_{t-h_1}^{t} x(s)^T S x(s) ds \\
+ h_1 \int_{-h_1}^{0} \int_{t}^{t} \dot{x}(s)^T R \dot{x}(s) ds \theta ds \\
+ \sum_{i=1}^{q} (h_2 - h_1) \int_{-h_2}^{0} \int_{t}^{t} \dot{x}(s)^T R_{ai} \dot{x}(s) ds \theta ds.
$$

Asymptotical stability condition depending of the derivative of $V(x(t), \dot{x}(t))$ along the system trajectories:

$$
V(x(t), \dot{x}(t)) > 0, \quad \dot{V}(x(t), \dot{x}(t)) < 0, \quad \text{for any} \quad x_t \neq 0, \\
\text{and} \quad V(x(0), \dot{x}(0)) = 0, \quad \dot{V}(x(0), \dot{x}(0)) = 0.
$$
2. Novel Control Schemes

😊 System Description & Assumptions;
😊 Robust Stability Conditions;
😊 Control Scheme 1 - Bilateral State Feedback Control Scheme

[B. Zhang et al., CCDC, 2011];

😊 Control Scheme 2 - Force-Reflecting Control Scheme;

😊 Control Scheme 3 - Force-Reflecting Proxy Control Scheme

[B. Zhang et al., ETFA, 2011];

😊 Results and Analysis;
General Teleoperation Structure:

Our Master/Slave Controller Solution:

- Lyapunov-Krasovskii functional (LKF);
- $H_\infty$ control;
- Linear Matrix Inequality (LMI) optimization;
General Teleoperation Structure:

From Property 1 - Assumption 1:
Linear master/slave systems

\[
\begin{align*}
\dot{x}_m(t) &= (A_m - B_m K^m_0)x_m(t) + B_m(F_m(t) + F_h(t)), \\
\dot{x}_s(t) &= (A_s - B_s K^s_0)x_s(t) + B_s(F_s(t) + F_e(t)),
\end{align*}
\]

\[x_m(t) = \dot{\theta}_m(t) \in \mathbb{R}^n, \quad x_s(t) = \dot{\theta}_s(t) \in \mathbb{R}^n; \quad K^m_0 \text{ & } K^s_0 : \text{supposed to be known};\]
**General Teleoperation Structure:**

From Property 2 - Assumption 2:
Internet/Ethernet/Wifi... $\Rightarrow \tau_1(t), \tau_2(t) \in [h_1, h_2], h_1 \geq 0$;
General Teleoperation Structure:

From Property 3 - Assumption 3:
Time-stamped data packets $\implies \hat{\tau}_1(t) = \tau_1(t), \hat{\tau}_2(t) = \tau_2(t)$;
2. Novel Control Schemes

- System Description & Assumptions;
- Robust Stability Conditions;
- Control Scheme 1 - Bilateral State Feedback Control Scheme;
  
  [B. Zhang et al., CCDC, 2011];
- Control Scheme 2 - Force-Reflecting Control Scheme;
- Control Scheme 3 - Force-Reflecting Proxy Control Scheme
  
  [B. Zhang et al., ETFA, 2011];
- Results and Analysis;
Linear Time Delay System:

\[
\begin{aligned}
\dot{x}(t) &= A_0 x(t) + \sum_{i=1}^{q} A_i x(t - \tau_i(t)), \\
x(t_0 + \theta) &= \phi(\theta), \quad \dot{x}(t_0 + \theta) = \dot{\phi}(\theta), \quad \theta \in [-h_2, 0].
\end{aligned}
\] (8)

Asymptotical Stability Theorem [E. Fridman, IMA Journal of Mathematical Control and Information, 2006]:

- \( P > 0, \ R > 0, \ S > 0, \ S_a > 0, \ R_{ai} > 0, \) and \( P_2, P_3, Y_1, Y_2, \)
  \( i = 1, 2, ..., q; \)
- LMI condition is feasible;
- Rate-independent asymptotically stable for time-varying delays \( \tau_i(t) \in [h_1, h_2], \ i = 1, 2, ..., q; \)
Asymptotical Stability Theorem :

\[
\Gamma_1 = \begin{pmatrix}
    \Gamma_{11}^1 & \Gamma_{12}^1 & \Gamma_{13}^1 & \cdots & \Gamma_{1q}^1 \\
    \Gamma_{21}^1 & \Gamma_{22}^1 & \Gamma_{23}^1 & \cdots & \Gamma_{2q}^1 \\
    \Gamma_{31}^1 & \Gamma_{32}^1 & \Gamma_{33}^1 & \cdots & \Gamma_{3q}^1 \\
    \cdots & \cdots & \cdots & \cdots & \cdots \\
    \Gamma_{q1}^1 & \Gamma_{q2}^1 & \Gamma_{q3}^1 & \cdots & \Gamma_{qq}^1 \\
\end{pmatrix} < 0, \\
\]

\[
\Gamma_{11}^1 = S + S_a - R + A_0^T P_2 + P_2^T A_0, \quad \Gamma_{12}^1 = P - P_2^T + A_0^T P_3, \\
\Gamma_{22}^1 = -P_3 - P_3^T + h_1^2 R + (h_2 - h_1)^2 \sum_{i=1}^{q} R_{ai}.
\]
**H_∞** Performance - Delay-Free with Perturbation:

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bw(t), \\
z(t) &= Cx(t).
\end{align*}
\]

\[
J(w) = \int_0^\infty (z(t)^Tz(t) - \gamma^2w(t)^Tw(t))dt < 0,
\]

\[
(sup_w(\frac{\|z(t)\|_2}{\|w(t)\|_2}) < \gamma).
\]

\[
\dot{V}(x(t)) + z(t)^Tz(t) - \gamma^2w(t)^Tw(t) < 0, \quad V(x(t)) = x(t)^TPx(t).
\]

**Robust Stability Theorem:**

- \(P > 0, P_2, P_3\), and a positive scalar \(\gamma > 0\);
- LMI condition is feasible (see manuscript);
- Asymptotically stable with **H_∞** performance \(J(w) < 0\).
**New Control Schemes for Bilateral Teleoperation under Asymmetric Communication Channels**

**CONTROL SCHEMES**

**Robust Stability Conditions**

**$H_{\infty}$ Performance - Delay-Free with Perturbation:**

\[
\begin{aligned}
\dot{x}(t) &= Ax(t) + Bw(t), \\
z(t) &= Cx(t).
\end{aligned}
\]

\(J(w) = \int_{0}^{\infty} (z(t)^T z(t) - \gamma^2 w(t)^T w(t)) dt < 0,\)

\(\left(\sup_{w} \left( \frac{\|z(t)\|_2}{\|w(t)\|_2} \right) < \gamma \right).\)

\[
\dot{V}(x(t)) + z(t)^T z(t) - \gamma^2 w(t)^T w(t) < 0, \quad V(x(t)) = x(t)^T P x(t).
\]

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New Control Schemes for Bilateral Teleoperation under Asymmetric Communication Channels

CONTROL SCHEMES

Robust Stability Conditions

$H_\infty$ Performance - Delay-Free with Perturbation :

\[
\begin{align*}
\dot{x}(t) &= A x(t) + B w(t), \\
z(t) &= C x(t).
\end{align*}
\]

\( J(w) = \int_{0}^{\infty} (z(t)^T z(t) - \gamma^2 w(t)^T w(t)) dt < 0, \)

\[ (\sup_{w} \left( \frac{\|z(t)\|_2}{\|w(t)\|_2} \right) < \gamma). \]  

\[
\dot{V}(x(t)) + z(t)^T z(t) - \gamma^2 w(t)^T w(t) < 0, \quad V(x(t)) = x(t)^T P x(t).
\]

Robust Stability Theorem :

- $P > 0$, $P_2$, $P_3$, and a positive scalar $\gamma > 0$;
- LMI condition is feasible (see manuscript);
- Asymptotically stable with $H_\infty$ performance $J(w) < 0$;
Robust Stability Condition - Teleoperation Case:

- LKF asymptotical stability condition with several time-varying delays
- \( H_\infty \) performance improvement condition without time-varying delays but with the perturbation
- Robust stability condition with several time-varying delays and the perturbation:

\[
\dot{x}(t) = A_0 x(t) + \sum_{i=1}^{q} A_i x(t - \tau_i(t)) + Bw(t), \quad z(t) = Cx(t), \quad x(t_0 + \theta) = \phi(\theta), \quad \dot{x}(t_0 + \theta) = \dot{\phi}(\theta), \quad \theta \in [-h_2, 0],
\]

\[
\dot{V}(x(t), \dot{x}(t)) + z(t)^T z(t) - \gamma^2 w(t)^T w(t) < 0.
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Robust Stability Condition - Teleoperation Case:

- LKF asymptotical stability condition with several time-varying delays
- \( H_\infty \) performance improvement condition without time-varying delays but with the perturbation

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z(t) &= C x(t), \\
x(t_0 + \theta) &= \phi(\theta), \quad \dot{x}(t_0 + \theta) = \dot{\phi}(\theta), \quad \theta \in [-h_2, 0], \\
\end{align*}
\]

\[
\dot{V}(x(t), \dot{x}(t)) + z(t)^T z(t) - \gamma^2 w(t)^T w(t) < 0.
\]
Robust Stability Condition - Teleoperation Case:

- LKF asymptotical stability condition with several time-varying delays

+ $H_\infty$ performance improvement condition without time-varying delays but with the perturbation

= Robust stability condition with several time-varying delays and the perturbation:

$$
\left\{
\begin{array}{l}
\dot{x}(t) = A_0 x(t) + \sum_{i=1}^{q} A_i x(t - \tau_i(t)) + B w(t), \\
z(t) = C x(t), \\
x(t_0 + \theta) = \phi(\theta), \quad \dot{x}(t_0 + \theta) = \dot{\phi}(\theta), \quad \theta \in [-h_2, 0],
\end{array}
\right.
$$

$$(13)$$

$$\dot{V}(x(t), \dot{x}(t)) + z(t)^T z(t) - \gamma^2 w(t)^T w(t) < 0.$$
2. Novel Control Schemes

- System Description & Assumptions;
- Robust Stability Conditions;
- Control Scheme 1 - Bilateral State Feedback Control Scheme;
  \[ B. Zhang et al., CCDC, 2011 \];
- Control Scheme 2 - Force-Reflecting Control Scheme;
- Control Scheme 3 - Force-Reflecting Proxy Control Scheme
  \[ B. Zhang et al., ETFA, 2011 \];
- Results and Analysis;
**Control Scheme 1 - Bilateral State Feedback Control Scheme**: 

Delayed state feedback $C_1$ & $C_2$:

\[
\begin{align*}
C_1 : \quad F_s(t) &= -K_1^1 \dot{\theta}_s(t - \hat{\tau}_1(t)) - K_1^2 \dot{\theta}_m(t - \tau_1(t)) \\
&\quad - K_1^3 (\theta_s(t - \hat{\tau}_1(t)) - \theta_m(t - \tau_1(t))), \\
C_2 : \quad F_m(t) &= -K_2^1 \dot{\theta}_s(t - \tau_2(t)) - K_2^2 \dot{\theta}_m(t - \hat{\tau}_2(t)) \\
&\quad - K_2^3 (\theta_s(t - \tau_2(t)) - \theta_m(t - \hat{\tau}_2(t))).
\end{align*}
\]  

$\hat{\tau}_1(t) = \tau_1(t)$ and $\hat{\tau}_2(t) = \tau_2(t)$ from Assumption 3;
Control Scheme 1 - Master/Slave Controller Design:

\[
\left\{
\begin{array}{l}
\dot{x}_{ms}(t) = (A_{ms} - B_{ms}K_0)x_{ms}(t) + B_{ms}u_{ms}(t) + B_{ms}w_{ms}(t), \\
u_{ms}(t) = -K^1_{ms}x_{ms}(t - \tau_1(t)) - K^2_{ms}x_{ms}(t - \tau_2(t)), \\
z_{ms}(t) = C_{ms}x_{ms}(t),
\end{array}
\right.
\]  
(15)

\[
x_{ms}(t) = \begin{pmatrix}
\dot{\theta}_s(t) \\
\dot{\theta}_m(t) \\
\theta_s(t) - \theta_m(t)
\end{pmatrix}, \quad u_{ms}(t) = \begin{pmatrix} F_s(t) \\
F_m(t) \end{pmatrix}, \quad w_{ms}(t) = \begin{pmatrix} F_e(t) \\
F_h(t) \end{pmatrix},
\]

\[
z_{ms}(t) = \begin{pmatrix} \theta_s(t) - \theta_m(t) \end{pmatrix},
\]

\[
A_{ms} = \begin{pmatrix} A_s & 0 & 0 \\
0 & A_m & 0 \\
1 & -1 & 0
\end{pmatrix}, \quad B_{ms} = \begin{pmatrix} B_s & 0 \\
0 & B_m \end{pmatrix} = \begin{pmatrix} B^1_{ms} & B^2_{ms} \end{pmatrix},
\]

\[
K_0 = \begin{pmatrix} K^s_0 & 0 & 0 \\
0 & K^m_0 & 0
\end{pmatrix}, \quad K^1_{ms} = \begin{pmatrix} K^1_1 & K^2_1 & K^3_1 \\
0 & K^2_1 & 0
\end{pmatrix}, \quad K^2_{ms} = \begin{pmatrix} 0 & K^1_2 & K^2_2 & K^3_2
\end{pmatrix},
\]

\[
C_{ms} = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}.
\]
Control Scheme 1 - Master/Slave Controller Design:

Transformation so to apply robust control condition:

\[
\begin{align*}
\dot{x}_{ms}(t) &= A_{0ms}^0 x_{ms}(t) + A_{1ms}^1 x_{ms}(t - \tau_1(t)) + A_{2ms}^2 x_{ms}(t - \tau_2(t)) \\
&\quad + B_{ms} w_{ms}(t), \\
z_{ms}(t) &= C_{ms} x_{ms}(t),
\end{align*}
\]

\[
A_{0ms}^0 = A_{ms} - B_{ms} K_0, \\
A_{1ms}^1 = -B_{ms} K_{ms}^1 = -B_{ms}^1 K_1, \\
A_{2ms}^2 = -B_{ms} K_{ms}^2 = -B_{ms}^2 K_2,
\]

\[
K_1 = \begin{pmatrix} K_1^1 & K_1^2 & K_1^3 \end{pmatrix}, \quad K_2 = \begin{pmatrix} K_2^1 & K_2^2 & K_2^3 \end{pmatrix}.
\]
Control Scheme 1 - Control Objectives:

- LKF: the system stability under the time-varying delays $\tau_1(t)$, $\tau_2(t) \in [h_1, h_2]$;
- $H_\infty$ control: the impact $\gamma$ of disturbances $w_{ms}(t)$ on $z_{ms}(t)$ ($\theta_s(t) - \theta_m(t)$);

Control Scheme 1 - Master/Slave Controller Design Theorem:

- $P > 0$, $R > 0$, $S > 0$, $S_a > 0$, $R_{a1} > 0$, $R_{a2} > 0$, $P_2$, $W_1$, $W_2$, $Y_1$, $Y_2$, and positive scalars $\gamma$ and $\xi$;
- LMI condition is feasible (see manuscript);
- Rate-independent asymptotically stable with $H_\infty$ performance $J(w) < 0$ for time-varying delays $\tau_1(t)$, $\tau_2(t) \in [h_1, h_2]$:

$$K_1 = W_1 P_2^{-1}, \quad K_2 = W_2 P_2^{-1}.$$ (21)
2. Novel Control Schemes

😊 System Description & Assumptions;
😊 Robust Stability Conditions;
😊 Control Scheme 1 - Bilateral State Feedback Control Scheme
  \[B. \text{Zhang et al., CCDC, 2011}\]
😊 Control Scheme 2 - Force-Reflecting Control Scheme;
😊 Control Scheme 3 - Force-Reflecting Proxy Control Scheme
  \[B. \text{Zhang et al., ETFA, 2011}\]
😊 Results and Analysis;
Control Scheme 2 - Force-Reflecting Control Scheme:

- \( \hat{F}_e(t) : F_m(t) = \hat{F}_e(t - \tau_2(t)) \);
- \( C \) with controller gain \( K^i, i = 1, 2, 3 \):

\[
C : \quad F_s(t) = -K^1 \dot{\theta}_s(t - \hat{\tau}_1(t)) - K^2 \dot{\theta}_m(t - \tau_1(t)) \\
- K^3 (\theta_s(t - \hat{\tau}_1(t)) - \theta_m(t - \tau_1(t))).
\]
Control Scheme 2 - Slave Controller Design:

\[
\begin{aligned}
\dot{x}_{ms}(t) &= (A_{ms} - B_{ms}K_0)x_{ms}(t) + B_{ms}u_{ms}(t) + B_{ms}w_{ms}(t), \\
u_{ms}(t) &= -K_{ms}x_{ms}(t - \tau_1(t)), \\
K_{ms} &= \begin{pmatrix} K^1 & K^2 & K^3 \\ 0 & 0 & 0 \end{pmatrix}.
\end{aligned}
\] (23)

\[
\begin{aligned}
\dot{x}_{ms}(t) &= A^0_{ms}x_{ms}(t) + A^1_{ms}x_{ms}(t - \tau_1(t)) + B_{ms}w_{ms}(t), \\
z_{ms}(t) &= C_{ms}x_{ms}(t), \\
A^1_{ms} &= -B_{ms}K_{ms} = -B^1_{ms}K, \\
K &= \begin{pmatrix} K^1 & K^2 & K^3 \end{pmatrix}.
\end{aligned}
\] (25)
2. Novel Control Schemes

- System Description & Assumptions;
- Robust Stability Conditions;
- Control Scheme 1 - Bilateral State Feedback Control Scheme;
  [B. Zhang et al., CCDC, 2011];
- Control Scheme 2 - Force-Reflecting Control Scheme;
- Control Scheme 3 - Force-Reflecting Proxy Control Scheme
  [B. Zhang et al., ETFA, 2011];
- Results and Analysis;
Control Scheme 3 - Force-Reflecting Proxy Control Scheme:

- From master to slave: $\dot{\theta}_m(t)/\theta_m(t)/\hat{F}_h(t)$, the position tracking;
- From slave to master: $F_m(t) = F_e(t - \tau_2(t))$, the force tracking;
Control Scheme 3 - Slave Controller Description:

\[ \dot{x}_p(t) = (A_m - B_m K_0^m)x_p(t) - B_m F_p(t) + B_m (\hat{F}_e(t - \hat{\tau}_1(t)) + \hat{F}_h(t - \tau_1(t))) \],

\[ x_p(t) = \dot{\theta}_p(t) \in \mathbb{R}^n. \]
**Control Scheme 3 - Slave Controller Description**:

\[
\mathbf{L} = \begin{pmatrix} L_1 & L_2 & L_3 \end{pmatrix} \implies \theta_p(t) \to \theta_m(t) : \\
\dot{F}_p(t) = \mathbf{L} \begin{pmatrix} \dot{\theta}_p(t-\hat{\tau}_1(t)) \\ \dot{\theta}_m(t-\tau_1(t)) \\ \theta_p(t-\hat{\tau}_1(t)) - \theta_m(t-\tau_1(t)) \end{pmatrix}. \\
\text{(27)}
\]
Control Scheme 3 - Slave Controller Description:

\[ \hat{\theta}_m(t) - \hat{\theta}_p(t) \]

\[ F_s(t) = -K \begin{pmatrix} \dot{\theta}_s(t) \\ \dot{\theta}_p(t) \\ \theta_s(t) - \theta_p(t) \end{pmatrix}. \quad (28) \]
Control Scheme 3 - Master-Proxy Synchronization:

\[ \begin{align*}
\dot{x}_{mp}(t) &= A^0_{mp} x_{mp}(t) + A^1_{mp} x_{mp}(t - \tau_1(t)) + B_{mp} w_{mp}(t), \\
z_{mp}(t) &= C_{mp} x_{mp}(t).
\end{align*} \tag{29} \]

\[ \begin{align*}
x_{mp}(t) &= \begin{pmatrix} \dot{\theta}_p(t) \\ \dot{\theta}_m(t) \\ \theta_p(t) - \theta_m(t) \end{pmatrix}, \\
w_{mp}(t) &= \begin{pmatrix} F_e(t - \tau_1(t)) + \hat{F}_h(t - \tau_1(t)) \\ F_m(t) + \hat{F}_h(t) \end{pmatrix}, \\
z_{mp}(t) &= \begin{pmatrix} \theta_p(t) - \theta_m(t) \end{pmatrix}.
\end{align*} \tag{30} \]
Control Scheme 3 - Master-Proxy Synchronization:

\[
A_{mp}^0 = \begin{pmatrix}
A_m - B_m K_0^m & 0 & 0 \\
0 & A_m - B_m K_0^m & 0 \\
1 & 0 & 0 \\
\end{pmatrix}, \quad B_{mp} = \begin{pmatrix}
B_m & 0 & 0 \\
0 & B_m & 0 \\
\end{pmatrix} = \begin{pmatrix}
B_{mp}^1 & B_{mp}^2 \\
\end{pmatrix},
\]

\[
C_{mp} = \begin{pmatrix}
0 & 0 & 1 \\
\end{pmatrix},
\]

\[
A_{mp}^1 = \begin{pmatrix}
-B_m L_1 & -B_m L_2 & -B_m L_3 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{pmatrix} = -B_{mp}^1 L.
\]

(31)
Control Scheme 3 - Proxy-Slave Synchronization:

\[
\begin{align*}
\dot{x}_{ps}(t) &= A_{ps}x_{ps}(t) + B_{ps}w_{ps}(t), \\
z_{ps}(t) &= C_{ps}x_{ps}(t),
\end{align*}
\]

\[
x_{ps}(t) = \begin{pmatrix} \theta_s(t) \\ \theta_p(t) \\ \theta_s(t) - \theta_p(t) \end{pmatrix}, \quad z_{ps}(t) = \begin{pmatrix} \theta_s(t) - \theta_p(t) \end{pmatrix},
\]

\[
w_{ps}(t) = \begin{pmatrix} F_e(t) \\ \hat{F}_e(t - \tau_1(t)) + \hat{F}_h(t - \tau_1(t)) - F_p(t) \end{pmatrix}.
\]
Control Scheme 3 - Proxy-Slave Synchronization:

\[ B_{ps} = \begin{pmatrix} B_s & 0 \\ 0 & B_m \end{pmatrix} \begin{pmatrix} B_{ps}^1 & B_{ps}^2 \\ 0 & 0 \end{pmatrix}, \quad C_{ps} = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}, \]

\[ A_{ps} = \begin{pmatrix} A_s - B_s K_0^s & -B_s K_1 & -B_s K_2 & -B_s K_3 \\ 0 & A_m - B_m K_0^m & 0 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix} \]

\[ = \begin{pmatrix} A_s - B_s K_0^s \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 & A_m - B_m K_0^m & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} + \begin{pmatrix} -B_s K_1 \\ -B_s K_2 \\ -B_s K_3 \end{pmatrix} \]

\[ = (A_{ps}^0 - B_{ps}^1 K). \]
Control Scheme 3 - Global Performance Analysis:

\[
\begin{align*}
\dot{x}_{mps}(t) &= A^0_{mps} x_{mps}(t) + A^1_{mps} x_{mps}(t - \tau_1(t)) + B_{mps} w_{mps}(t), \\
z_{mps}(t) &= C_{mps} x_{mps}(t),
\end{align*}
\]

(35)

\[
x_{mps}(t) = \begin{pmatrix}
\dot{\theta}_s(t) \\
\dot{\theta}_p(t) \\
\dot{\theta}_m(t) \\
\theta_s(t) - \theta_p(t) \\
\theta_p(t) - \theta_m(t)
\end{pmatrix},
\quad
w_{mps}(t) = \begin{pmatrix}
\hat{F}_e(t) \\
\hat{F}_h(t - \tau_1(t)) + \hat{F}_h(t - \tau_1(t))
\end{pmatrix},
\quad
z_{mps}(t) = \begin{pmatrix}
\theta_s(t) - \theta_p(t) \\
\theta_p(t) - \theta_m(t)
\end{pmatrix}.
\]

(36)

\[
A^0_{mps} = \begin{pmatrix}
A_s - B_s G^s_0 - B_s K_1 & -B_s K_2 & 0 & -B_s K_3 & 0 \\
0 & A_m - B_m G^m_0 & 0 & 0 & 0 \\
0 & 0 & A_m - B_m G^m_0 & 0 & 0 \\
1 & -1 & 0 & -1 & 0 \\
0 & 1 & 0 & 0 & 0
\end{pmatrix},
\]

(37)

\[
A^1_{mps} = \begin{pmatrix}
0 & -B_m L_1 & -B_m L_2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{pmatrix},
\quad
B_{mps} = \begin{pmatrix}
B_s & 0 & 0 \\
0 & B_m & 0 \\
0 & 0 & B_m
\end{pmatrix},
\quad
C_{mps} = \begin{pmatrix}
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix}.
\]
Conclusions:

😊 Control Scheme 1 - Bilateral state feedback control scheme;
😊 Control Scheme 2 - Force-reflecting control scheme;
😊 Control Scheme 3 - Force-reflecting proxy control scheme;

<table>
<thead>
<tr>
<th>Control Schema</th>
<th>Time Delays</th>
<th>Position Tracking</th>
<th>Force Tracking</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Constant</td>
<td>Time-varying</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>2</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>3</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

😊 Force estimation/measure: 2 & 3;
😊 Better performance, but additional computation load: 3;
2. Novel Control Schemes

- System Description & Assumptions;
- Robust Stability Conditions;
- Control Scheme 1 - Bilateral State Feedback Control Scheme [B. Zhang et al., CCDC, 2011];
  
  [B. Zhang et al., CCDC, 2011];
- Control Scheme 2 - Force-Reflecting Control Scheme;
- Control Scheme 3 - Force-Reflecting Proxy Control Scheme
  
  [B. Zhang et al., ETFA, 2011];
- Results and Analysis;
Simulation Conditions:

- Maximum amplitude of time-varying delays: 0.2s;
- Master, proxy and slave models: 1/s, 1/s and 2/s; Poles: $[-100.0]$.
- $K_0^m = 100$, $K_0^s = 50$;

\[
K_1 = \begin{pmatrix} -0.1870 & -0.0368 & 65.0846 \end{pmatrix},
\]
\[
K_2 = \begin{pmatrix} 0.4419 & 0.0813 & -153.8704 \end{pmatrix}, \quad \gamma_{C_1/C_2}^{\min} = 0.0123. \tag{38}
\]

\[
L = \begin{pmatrix} -1.4566 & 0.1420 & 282.482 \end{pmatrix}, \quad \gamma_{\min}^L = 0.0081,
\]
\[
K = \begin{pmatrix} -29.9635 & -3.6393 & 618.536 \end{pmatrix}, \quad \gamma_{\min}^K = 0.0075. \tag{39}
\]

Global stability of the system is verified with $\gamma_{\min}^q = 0.0062$;
Abrupt Tracking Motion:

- Bilateral State Feedback:
  - Master and Slave positions for force feedback $F_h(t) = 10N$ and $F_h(t) = 0N$.

- Force-Reflecting Proxy:
  - Master and Slave positions for force feedback $F_h(t) = 10N$ and $F_h(t) = 0N$.

Passivity from [Y. Ye et al., CDC, 2009]

LKF from [C.-C. Hua et al., IEEE Trans. Robot., 2010]
Abrupt Tracking Motion:

![Graphs showing position and simulation time for different force scenarios.](image-url)
Wall Contact Motion:

😊 Hard wall: the stiffness $K_e = 30kN/m$, the position $x = 1.0m$;

😊 Our aims:

1. When the slave robot reaches the wall, the master robot can stop as quickly as possible;

2. When the slave robot returns after hitting the wall ($F_e(t) = 0$), the system must restore the position tracking between the master and the slave;

3. When the slave contacts the wall, the force tracking from the slave to the master can be assured;
Wall Contact Motion:
Wall Contact Motion:

![Graphs showing position and simulation time for different control schemes.](image)
Wall Contact Motion:

Force response in wall contact motion \( F_m(t) ; \hat{F}_e(t) \).
Wall Contact Motion under Large Delays $h_2 = 1.0\, s$:

- **Bilateral State Feedback**
- **Force-Reflecting Proxy**
- **LKF from [C.-C. Hua et al., IEEE Trans. Robot., 2010]**
3. Robustness Aspects

😊 Discrete-Time System [B. Zhang et al., TDS, 2012] :

- Discrete-Time Approach;
- Slave Controller Design;
- Results and Analysis;

😊 Linear Parameter-Varying System (LPV) :

- Polytopic-type uncertainties [B. Zhang et al., TDS, 2012];
- Norm-bounded uncertainties [B. Zhang et al., ROCOND, 2012];
- Results and Analysis;
Discrete-Time System:

\[
\begin{align*}
    x(k+1) &= \sum_{i=0}^{q} A_i x(k - \tau_i(k)) + Bw(k), \\
    z(k) &= Cx(k).
\end{align*}
\]  

(40)

Delay-Free Case:

\[
\begin{align*}
    x(k+1) &= A_0 x(k) + Bw(k), \\
    z(k) &= Cx(k).
\end{align*}
\]  

(41)
Robust Stability Condition:

- Discrete LKF, $y(k) = x(k + 1) - x(k)$:

\[
V(x(k)) = x(k)^T P x(k) + \sum_{i=k-h_2}^{k-1} x(i)^T S_a x(i) + \sum_{i=k-h_1}^{k-1} x(i)^T S x(i) + h_1 \sum_{i=-h_1}^{-1} \sum_{j=k+i}^{k-1} y(j)^T R y(j) + \sum_{i=1}^{q} (h_2 - h_1) \sum_{j=-h_2}^{-1} \sum_{l=k+j}^{k-1} y(l)^T R_{ai} y(l).
\]

(42)

- $H_\infty$ control: $J(w) = \sum_{i=0}^{\infty} [z(k)^T z(k) - \gamma^2 w(k)^T w(k)] < 0$

- Rate-independent asymptotically stable and $H_\infty$ performance $J(w) < 0$ for time-varying delays $\tau_i(k) \in [h_1, h_2]$, $h_2 \geq h_1 \geq 0$, $i = 1, 2, \ldots, q$.
3. Robustness Aspects

😄 **Discrete-Time System** [B. Zhang et al., *TDS*, 2012]:
   - *Discrete-Time Approach*;
   - *Slave Controller Design*;
   - *Results and Analysis*;

😊 **Linear Parameter-Varying System (LPV)**:
   - *Polytopic-type uncertainties* [B. Zhang et al., *TDS*, 2012];
   - *Norm-bounded uncertainties* [B. Zhang et al., *ROCOND*, 2012];
   - *Results and Analysis*;
System Description:

\[
\begin{align*}
(\Sigma^d_m) & \quad x_m(k + 1) = (A_{md} - B_{md}K_{md}^0)x_m(k) + B_{md}(F_m(k) + F_h(k)), \\
(\Sigma^d_s) & \quad x_s(k + 1) = (A_{sd} - B_{sd}K_{sd}^0)x_s(k) + B_{sd}(F_s(k) + F_e(k)), \\
(\Sigma^d_p) & \quad x_p(k + 1) = (A_{md} - B_{md}K_{md}^0)x_p(k) - B_{md}F_p(k) \\
& \quad + B_{md}(\hat{F}_e(k - \hat{\tau}_1(k)) + \hat{F}_h(k - \tau_1(k))).
\end{align*}
\]
Slave Controller Design - Master-Proxy Synchronization:

\[ L_d = \begin{pmatrix} L_{d1} & L_{d2} & L_{d3} \end{pmatrix} : \]

\[ F_p(k) = L_d \begin{pmatrix} \dot{\theta}_p(k - \hat{\tau}_1(k)) \\ \dot{\theta}_m(k - \tau_1(k)) \\ \theta_p(k - \hat{\tau}_1(k)) - \theta_m(k - \tau_1(k)) \end{pmatrix} . \quad (45) \]

\[ (\Sigma_{d_{mp}}) \]

\[ x_{mp}(k + 1) = A^0_{mpd} x_{mp}(k) + A^1_{mpd} x_{mp}(k - \tau_1(k)) + B_{mpd} w_{mp}(k), \]

\[ z_{mp}(k) = C_{mpd} x_{mp}(k), \]

\[ A^1_{mpd} \implies L_d ; \]
Slave Controller Design - Proxy-Slave Synchronization:

\( K_d = \begin{pmatrix} K_{d1} & K_{d2} & K_{d3} \end{pmatrix} \):

\[ F_s(k) = -K_d \begin{pmatrix} \dot{\theta}_s(k) \\ \dot{\theta}_p(k) \\ \theta_s(k) - \theta_p(k) \end{pmatrix}. \quad (47) \]

\[
\begin{align*}
(\Sigma_{ps}^d) \left\{ 
\begin{array}{l}
x_{ps}(k + 1) = A_{psd}x_{ps}(k) + B_K F_s(k) + B_{psd} w_{ps}(k), \\
z_{ps}(k) = C_{psd}x_{ps}(k),
\end{array}
\right.
\end{align*}
\]

\[
\downarrow
\]

\[
\begin{align*}
(\Sigma_{ps}^d) \left\{ 
\begin{array}{l}
x_{ps}(k + 1) = (A_{psd} - B_K K_d)x_{ps}(k) + B_{psd} w_{ps}(k), \\
z_{ps}(k) = C_{psd}x_{ps}(k).
\end{array}
\right.
\end{align*}
\]

(49)
Slave Controller Design - Global Performance Analysis:

\[
\begin{align*}
(\Sigma^d_{mps}) & \left\{ \begin{array}{l}
x(k+1) = A_{mps}^d x(k) + B_{mps}^K F_s(k) - B_{mps}^L F_p(k) + B_{mps} w(k), \\
z(k) = C_{mps}^d x(k),
\end{array} \right. \\
F_s(k) &= -\bar{K}_d x(k) = -\begin{pmatrix} K_{d1} & K_{d2} & 0 & K_{d3} & 0 \end{pmatrix} x(k), \\
F_p(k) &= \bar{L}_d x(k - \tau_1(k)) = \begin{pmatrix} 0 & L_{d1} & L_{d2} & 0 & L_{d3} \end{pmatrix} x(k - \tau_1(k)), \\
B_{mps}^K &= \begin{pmatrix} B_{sd} \\ 0 \\ 0 \\ 0 \end{pmatrix}, \\
B_{mps}^L &= \begin{pmatrix} 0 \\ B_{md} \\ 0 \\ 0 \end{pmatrix}, \\
\Downarrow \\
(\Sigma^d_{mps}) & \left\{ \begin{array}{l}
x(k+1) = A_0^d x(k) + A_1^d x(k - \tau_1(k)) + B_{mps} w(k), \\
z(k) = C_{mps}^d x(k),
\end{array} \right.
\end{align*}
\]

\[A_0^d = A_{mps} - B_{mps}^K \bar{K}_d, \quad A_1^d = -B_{mps}^L \bar{L}_d.\]
3. Robustness Aspects

😊 Discrete-Time System [B. Zhang et al., TDS, 2012]:

➤ Discrete-Time Approach;
➤ Slave Controller Design;
➤ Results and Analysis;

😊 Linear Parameter-Varying System (LPV):

➤ Polytopic-type uncertainties [B. Zhang et al., TDS, 2012];
➤ Norm-bounded uncertainties [B. Zhang et al., ROCOND, 2012];
➤ Results and Analysis;
Simulation Conditions:

😊 $T = 0.001\text{s}, h_1 = 1, h_2 = 100$ (in continuous-time domain, $h_1 = 0.001\text{s}, h_2 = 0.1\text{s}$);

😊

$L_d = \begin{pmatrix} 4.6815 & -5.1390 & 540.7828 \end{pmatrix}, \quad \gamma_{min}^{L_d} = 0.0051,$


\[ K_d = \begin{pmatrix} 273 & -127 & 10961 \end{pmatrix}, \quad \gamma_{min}^{K_d} = 2.9568 \times 10^{-4}, \quad (53) \]

\[ \gamma_{min}^g = 0.0327. \]
Abrupt Tracking Motion:

Continuous-Time

Discrete-Time

\[ F_m(t) = 10N \quad F_s(t) = 0N \]

\[ F_m(k) = 10N \quad F_s(k) = 0N \]
Abrupt Tracking Motion:

\[ F_h(t) = 10N \quad \text{and} \quad F_h(t) = 0N \]

Continuous-Time

\[ F_h(k) = 10N \quad \text{and} \quad F_h(k) = 0N \]

Discrete-Time
Abrupt Tracking Motion:

![Graphs showing continuous-time and discrete-time simulation results.](image-url)
Wall Contact Motion:
Wall Contact Motion:
3. Robustness Aspects

😊 Discrete-Time System [B. Zhang et al., TDS, 2012]:

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😊 Linear Parameter-Varying System (LPV):

- *Polytopic-type uncertainties* [B. Zhang et al., TDS, 2012];
- *Norm-bounded uncertainties* [B. Zhang et al., ROCOND, 2012];
- *Results and Analysis*;
System Description:

\[(\Sigma_m)\dot{x}_m(t) = (A_m(\rho_m(t)) - B_m(\rho_m(t))K_0^m)x_m(t) + B_m(\rho_m(t))(F_m(t) + F_h(t)),\]
\[(\Sigma_s)\dot{x}_s(t) = (A_s(\rho_s(t)) - B_s(\rho_s(t))K_0^s)x_s(t) + B_s(\rho_s(t))(F_s(t) + F_e(t)),\]
\[(\Sigma_p)\dot{x}_p(t) = (A_m(\rho_p(t)) - B_m(\rho_p(t))K_0^m)x_p(t) + B_m(\rho_p(t))(\hat{F}_e(t - \tau_1(t)) + \hat{F}_h(t - \tau_1(t)) - F_p(t)),\]

\[\begin{bmatrix} A_m(\rho_m(t)), B_m(\rho_m(t)) \end{bmatrix} = \sum_{j=1}^{N} \rho_{mj}(t)[A_{mj}, B_{mj}], \quad \begin{bmatrix} A_s(\rho_s(t)), B_s(\rho_s(t)) \end{bmatrix} = \sum_{j=1}^{N} \rho_{sj}(t)[A_{sj}, B_{sj}],\]

\[\begin{bmatrix} A_m(\rho_p(t)), B_m(\rho_p(t)) \end{bmatrix} = \sum_{j=1}^{N} \rho_{pj}(t)[A_{mj}, B_{mj}].\]
Controller Design - Problem 1:

\[ K^0_m \text{ & } K^0_s : \] robust stability \( w.r.t \) polytopic-type uncertainties;

Controller Design - Problem 2:

\[ L \text{ & } K : \] stability \& position/force tracking \( w.r.t \) time-varying delays \& polytopic-type uncertainties;

Robust Stability Theorem [E. Fridman, IMA Journal of Mathematical Control and Information, 2006]:

\[
\begin{align*}
\text{Q} & \quad P > 0, \; R > 0, \; S > 0, \; S_a > 0, \; R_{ai} > 0, \; P_2, \; P_3, \; Y_1, \; Y_2, \\
\text{Q} & \quad i = 1, 2, \ldots, q, \text{ and a positive scalar } \gamma > 0; \\
\text{Q} & \quad \text{N LMI conditions are feasible (see manuscript);} \\
\text{Q} & \quad \text{Rate-independent asymptotically stable with } H_\infty \text{ performance } \\
& J(w) < 0 \text{ for time-varying delays } \tau_i(t) \in [h_1, h_2], \; i = 1, 2, \ldots, q;
\end{align*}
\]
Problem 2 Slave Controller Design

Master-proxy synchronization:

\[
\begin{align*}
    \dot{x}_{mp}(t) &= A_{mp}^0(\rho_{mp}(t))x_{mp}(t) + A_{mp}^1(\rho_{mp}(t))x_{mp}(t - \tau_1(t)) + B_{mp}(\rho_{mp}(t))w_{mp}(t), \\
    z_{mp}(t) &= C_{mp}x_{mp}(t),
\end{align*}
\]

\(K_m^0 \& K_s^0 \implies A_{mp}^0(\rho_{mp}(t)); \quad A_{mp}^1(\rho_{mp}(t)) \implies L;\)

Proxy-slave synchronization:

\[
\begin{align*}
    \dot{x}_{ps}(t) &= A_{ps}(\rho_{ps}(t))x_{ps}(t) + B_{ps}(\rho_{ps}(t))w_{ps}(t), \\
    z_{ps}(t) &= C_{ps}x_{ps}(t),
\end{align*}
\]

\(A_{ps}(\rho_{ps}(t)) \implies K;\)

Global performance analysis with \(K_m^0 \& K_s^0, L \& K:\)

\[
\begin{align*}
    \dot{x}_{mps}(t) &= A_{mps}^0(\rho_{mps}(t))x_{mps}(t) + A_{mps}^1(\rho_{mps}(t))x_{mps}(t - \tau_1(t)) \\
    &\quad + B_{mps}(\rho_{mps}(t))w_{mps}(t), \\
    z_{mps}(t) &= C_{mps}x_{mps}(t).
\end{align*}
\]
Remark - Nonlinear Systems:

\[(\Sigma_m) \quad M_m(\theta_m)\ddot{\theta}_m(t) + C_m(\theta_m, \dot{\theta}_m)\dot{\theta}_m(t) + g_m(\theta_m) = F_h(t) + F_m(t),\]
\[(\Sigma_p) \quad M_m(\theta_p)\ddot{\theta}_p(t) + C_m(\theta_p, \dot{\theta}_p)\dot{\theta}_p(t) + g_m(\theta_p) = \hat{F}_e(t - \tau_1(t)) + \hat{F}_h(t - \tau_1(t)) - F_p(t),\]
\[(\Sigma_s) \quad M_s(\theta_s)\ddot{\theta}_s(t) + C_s(\theta_s, \dot{\theta}_s)\dot{\theta}_s(t) + g_s(\theta_s) = F_e(t) + F_s(t),\]

\[58\]

\[\Downarrow\]

\[(\bar{\Sigma}_m) \quad \ddot{\theta}_m(t) = A_m(\theta_m, \dot{\theta}_m)\dot{\theta}_m(t) + B_m(\theta_m)(F_h(t) + F_m(t) - g_m(\theta_m)),\]
\[(\bar{\Sigma}_p) \quad \ddot{\theta}_p(t) = A_m(\theta_p, \dot{\theta}_p)\dot{\theta}_p(t) + B_m(\theta_p)(\hat{F}_e(t - \tau_1(t)) + \hat{F}_h(t - \tau_1(t)) - F_p(t) - g_m(\theta_p)),\]
\[(\bar{\Sigma}_s) \quad \ddot{\theta}_s(t) = A_s(\theta_s, \dot{\theta}_s)\dot{\theta}_s(t) + B_s(\theta_s)(F_e(t) + F_s(t) - g_s(\theta_s)),\]

\[59\]

\[A_m(\theta_m, \dot{\theta}_m) = -M_m^{-1}(\theta_m)C_m(\theta_m, \dot{\theta}_m), \quad B_m(\theta_m) = M_m^{-1}(\theta_m),\]
\[A_m(\theta_p, \dot{\theta}_p) = -M_m^{-1}(\theta_p)C_m(\theta_p, \dot{\theta}_p), \quad B_m(\theta_p) = M_m^{-1}(\theta_p),\]
\[A_s(\theta_s, \dot{\theta}_s) = -M_s^{-1}(\theta_s)C_s(\theta_s, \dot{\theta}_s), \quad B_s(\theta_s) = M_s^{-1}(\theta_s).\]

\[60\]
3. Robustness Aspects

😊 Discrete-Time System [B. Zhang et al., TDS, 2012]:

► Discrete-Time Approach;
► Slave Controller Design;
► Results and Analysis;

😊 Linear Parameter-Varying System (LPV):

► Polytopic-type uncertainties [B. Zhang et al., TDS, 2012];
► Norm-bounded uncertainties [B. Zhang et al., ROCOND, 2012];
► Results and Analysis;
System Description:

\[
\begin{align*}
(\Sigma_m) & \quad \dot{x}_m(t) = ((A_m + \Delta A_m(t)) - (B_m + \Delta B_m(t))K^0_m)x_m(t) \\
& \quad \quad + (B_m + \Delta B_m(t))(F_m(t) + F_h(t)), \\
(\Sigma_s) & \quad \dot{x}_s(t) = ((A_s + \Delta A_s(t)) - (B_s + \Delta B_s(t))K^0_s)x_s(t) \\
& \quad \quad + (B_s + \Delta B_s(t))(F_s(t) + F_e(t)), \\
(\Sigma_p) & \quad \dot{x}_p(t) = ((A_m + \Delta A_p(t)) - (B_m + \Delta B_p(t))K^0_m)x_p(t) \\
& \quad \quad - (B_m + \Delta B_p(t))F_p(t) \\
& \quad \quad + (B_m + \Delta B_p(t))(\hat{F}_e(t - \hat{\tau}_1(t)) + \hat{F}_h(t - \tau_1(t))), \\
\end{align*}
\]

\[i = \{m, s, p\} : \quad \Delta A_i(t) = G_i \Delta(t) D_i, \quad \Delta B_i(t) = H_i \Delta(t) E_i, \quad \Delta(t)^T \Delta(t) \leq I.\]
**System Description:**

\[
\begin{align*}
(\Sigma_m) \quad \dot{x}_m(t) &= ((A_m + \Delta A_m(t)) - (B_m + \Delta B_m(t))K_m^0)x_m(t) \\
&+ (B_m + \Delta B_m(t))(F_m(t) + F_h(t)), \\
(\Sigma_s) \quad \dot{x}_s(t) &= ((A_s + \Delta A_s(t)) - (B_s + \Delta B_s(t))K_s^0)x_s(t) \\
&+ (B_s + \Delta B_s(t))(F_s(t) + F_e(t)), \\
(\Sigma_p) \quad \dot{x}_p(t) &= ((A_m + \Delta A_p(t)) - (B_m + \Delta B_p(t))K_m^0)x_p(t) \\
&- (B_m + \Delta B_p(t))F_p(t) \\
&+ (B_m + \Delta B_p(t))(\hat{F}_e(t - \hat{\tau}_1(t)) + \hat{F}_h(t - \tau_1(t))),
\end{align*}
\]

\(i = \{m, s, p\}: \quad \Delta A_i(t) = G_i\Delta(t)D_i, \quad \Delta B_i(t) = H_i\Delta(t)E_i, \quad \Delta(t)^T\Delta(t) \leq I. \tag{64}\)

**Slave Controller Design Solution:**

Transformation from norm-bounded uncertain system to linear time-delay system;
Problem 2 Slave Controller Design - Master-Proxy Synchronization:

\[
\begin{aligned}
\dot{x}_{mp}(t) &= (A_{0mp} + \Delta A_{0mp}(t))x_{mp}(t) + (A_{1mp} + \Delta A_{1mp}(t))x_{mp}(t - \tau_1(t)) \\
&\quad + (B_{mp} + \Delta B_{mp}(t))w_{mp}(t), \\
z_{mp}(t) &= C_{mp}x_{mp}(t),
\end{aligned}
\]

\[
K_m^0 \& K_s^0 \implies A_{0mp} + \Delta A_{0mp}(t) ;
\]

\[
\varphi_p(t) = (\Delta A_p(t) - \Delta B_p(t)K_m^0)\dot{\theta}_p(t) + \Delta B_p(t)(\hat{F}_e(t - \hat{\tau}_1(t)) + \hat{F}_h(t - \tau_1(t))),
\]

\[
\varphi_m(t) = (\Delta A_m(t) - \Delta B_m(t)K_m^0)\dot{\theta}_m(t) + \Delta B_m(t)(F_m(t) + F_h(t)),
\]

\[
\mu_p(t) = -\Delta B_p(t)Lx_{mp}(t - \tau_1(t)).
\]

\[
\begin{aligned}
\dot{x}_{mp}(t) &= A_{0mp}x_{mp}(t) + A_{1mp}x_{mp}(t - \tau_1(t)) + \tilde{B}_{mp}\tilde{w}_{mp}(t), \\
z_{mp}(t) &= C_{mp}x_{mp}(t),
\end{aligned}
\]

\[
\tilde{w}(t) = \begin{pmatrix}
B_m\hat{F}_e(t - \hat{\tau}_1(t)) + B_m\hat{F}_h(t - \tau_1(t)) + \varphi_p(t) + \mu_p(t) \\
B_mF_m(t) + B_mF_h(t) + \varphi_m(t)
\end{pmatrix},
\]

\[
A_{1mp} \implies L ;
\]
Slave Controller Design - Proxy-Slave Synchronization:

\[
\begin{align*}
\dot{x}_{ps}(t) &= (A_{ps} + \Delta A_{ps}(t))x_{ps}(t) + (B_{ps} + \Delta B_{ps}(t))w_{ps}(t), \\
z_{ps}(t) &= C_{ps}x_{ps}(t),
\end{align*}
\]

\[A_{ps} + \Delta A_{ps}(t) \implies K;\]

\[
\varphi_s(t) = (\Delta A_s(t) - \Delta B_s(t)K_s^0)\dot{\theta}_s(t) + \Delta B_s(t)F_e(t),
\]

\[
\mu_s(t) = -\Delta B_{ps}^1(t)Kx_{ps}(t).
\]

\[
\begin{align*}
\dot{x}_{ps}(t) &= A_{ps}x_{ps}(t) + \tilde{B}_{ps}\tilde{w}_{ps}(t), \\
z_{ps}(t) &= C_{ps}x_{ps}(t),
\end{align*}
\]

\[
\tilde{w}_{ps}(t) = \left(\begin{array}{c}
B_sF_e(t) + \varphi_s(t) + \mu_s(t) \\
B_mF_e(t - \hat{\tau}_1(t)) + B_mF_h(t - \tau_1(t)) - B_mF_p(t) + \varphi_p(t) + \mu_p(t)
\end{array}\right), \quad \tilde{B}_{ps} = \left(\begin{array}{c}
1 \\
0 \\
0 \\
0
\end{array}\right),
\]

\[A_{ps} \implies K;\]
Slave Controller Design - Global Performance Analysis:

\[
\begin{align*}
\dot{x}_{mps}(t) &= (A_{mps}^0 + \Delta A_{mps}^0(t))x_{mps}(t) + (A_{mps}^1 + \Delta A_{mps}^1(t))x_{mps}(t - \tau_1(t)) + (B_{mps} + \Delta B_{mps}(t))w_{mps}(t), \\
z_{mps}(t) &= C_{mps}x_{mps}(t).
\end{align*}
\]  

(73)

\[
\begin{align*}
\dot{x}_{mps}(t) &= A_{mps}^0x_{mps}(t) + A_{mps}^1x_{mps}(t - \tau_1(t)) + \tilde{B}_{mps}\tilde{w}_{mps}(t), \\
z_{mps}(t) &= C_{mps}x_{mps}(t),
\end{align*}
\]  

(74)

\[
\tilde{w}_{mps}(t) = \begin{pmatrix} B_s F_e(t) + \varphi_s(t) \\ B_m \hat{F}_e(t - \tau_1(t)) + B_m \hat{F}_h(t - \tau_1(t)) + \varphi_p(t) \\ B_m F_m(t) + B_m F_h(t) + \varphi_m(t) \end{pmatrix}, \quad \tilde{B}_{mps} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}.
\]  

(75)
3. Robustness Aspects

😊 Discrete-Time System  [B. Zhang et al., TDS, 2012] :

- *Discrete-Time Approach*;
- *Slave Controller Design*;
- *Results and Analysis*;

😊 Linear Parameter-Varying System (LPV) :

- *Polytopic-type uncertainties*  [B. Zhang et al., TDS, 2012] ;
- *Norm-bounded uncertainties*  [B. Zhang et al., ROCOND, 2012] ;
- *Results and Analysis* ;
Simulation Conditions:

\[ polytopic - type: \quad A_m(\rho_m(t)) = A_s(\rho_s(t)) = 0, \]
\[ B_m(\rho_m(t)) = B_s(\rho_s(t)) = \frac{1}{\rho(t)}, \quad \rho(t) \in [0.5, 1], \]  
\[ \text{(76)} \]

\[ norm - bounded - type: \quad A_m = A_s = 0, \quad G_i = D_i = 0, \]
\[ B_m = B_s = 1.5, \quad H_i = 0.5, \quad E_i = 1, \quad i = \{m, p, s\}. \]

\[ polytopic - type: \quad K^0_m = 2.6585, \quad K^0_s = 2.6585, \]  
\[ \text{(77)} \]

\[ norm - bounded - type: \quad K^0_m = 9.5117, \quad K^0_s = 3.3812. \]

\[ polytopic - type: \quad L = \begin{pmatrix} 1.3218 & -1.3219 & 6.3602 \end{pmatrix}, \quad \gamma^L_{\min} = 0.4436, \]
\[ K = \begin{pmatrix} 20.4799 & -21.2537 & 575.2051 \end{pmatrix}, \quad \gamma^K_{\min} = 0.0164, \]
\[ \gamma^g_{\min} = 0.4595, \]  
\[ \text{(78)} \]

\[ norm - bounded - type: \quad L = \begin{pmatrix} 1.6218 & -1.6264 & 29.5012 \end{pmatrix}, \quad \gamma^L_{\min} = 0.05, \]
\[ K = \begin{pmatrix} 16.4993 & -12.0059 & 434.4988 \end{pmatrix}, \quad \gamma^K_{\min} = 0.0072, \]
\[ \gamma^g_{\min} = 0.0376. \]
Abrupt Tracking Motion:
Wall Contact Motion:

![Graphs showing Wall Contact Motion for Polytopic-Type and Norm-Bounded-Type](image-url)
4. Experimentations

😊 Experimental Test-Bench;
😊 Force Estimation;
😊 Results and Analysis;
Experimental Test-Bench:

Master, Phantom Premium 1.0A:

$$\ddot{\theta}_m(t) = -\frac{\dot{\theta}_m(t)}{\tau_m} + \frac{K_m}{\tau_m} u_m(t), \tag{79}$$

$$\tau_m = 0.448 \text{s}, \quad K_m = 0.0176 \text{s/kg};$$
EXPERIMENTATIONS

Experimental Test-Bench:

\[ \dot{\theta}_s(t) = -\frac{\dot{\theta}_s(t)}{\tau_s} + \frac{K_s}{\tau_s} u_s(t) - F_s \text{sign}(\dot{\theta}_s(t)), \]  

\( \tau_s = 0.32 s, \)  \( K_s = 1.85 s/kg, \)  \( F_s = 0.30 m/s^2; \)
Experimental Test-Bench:

Networks: UDP;
4. Experimentations

😊 Experimental Test-Bench;
😊 Force Estimation;
😊 Results and Analysis;
Force Estimation $F_h(t)$:

\[
\dot{x}_m(t) = (A_m - B_m K_m^0) x_m(t) + B_m (F_m(t) + F_h(t)). \tag{81}
\]

\[
F_h^{(n+1)}(t) = 0 \text{ for some } n:
\]

\[
\dot{\varepsilon}_h(t) = A_h \varepsilon_h(t) + B_h F_m(t), \tag{82}
\]

\[
\varepsilon_h(t) = \begin{pmatrix} x_m(t) \\ F_h(t) \\ \vdots \\ F_h^{(n)}(t) \end{pmatrix}, \quad A_h = \begin{pmatrix} A_m - B_m K_m^0 & B_m & 0 & \cdots & 0 \\ 0 & 0 & I & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & I \end{pmatrix}, \quad B_h = \begin{pmatrix} B_m \\ 0 \\ \vdots \\ 0 \end{pmatrix}. \tag{83}
\]

\[
y_h(t) = x_m(t) = C_h \varepsilon_h(t):
\]

\[
\dot{\hat{\varepsilon}}_h(t) = A_h \hat{\varepsilon}_h(t) + B_h F_m(t) + L_h (y_h(t) - \hat{y}_h(t)). \tag{84}
\]

\[
e_h(t) = \varepsilon_h(t) - \hat{\varepsilon}_h(t):
\]

\[
\dot{e}_h(t) = (A_h - L_h C_h) e_h(t). \tag{85}
\]
Force Estimation $F_h(t)$:

$n = 1$, the eigenvalues of Luenberger observer as $[-11, -10, -9]$;
4. Experimentations

😊 Experimental Test-Bench;
😊 Force Estimation;
😊 Results and Analysis;
 Abrupt Tracking Motion :

\[ h_2 = 0.3s; \]
Wall Contact Motion:
Wall Contact Motion under Constant $F_h(t)$:
5. Conclusions & Perspectives
Conclusions:

3 novel control schemes ($3 \geq 2 \geq 1$):

1. Bilateral state feedback control scheme (Position/Velocity-Position/Velocity);
2. Force-reflecting control scheme (Position/Velocity-Force);
3. Force-reflecting proxy control scheme (Position/Velocity/Force-Force);

Formal proofs:

1. Stability (LKF);
2. Performance ($H_\infty$ control);
3. Time-varying (large) delays;
4. Uncertainties (polytopic or norm bounded);
5. Continuous or discrete time;

LMI design of the controllers;

Simulations and experimentations:

1. Abrupt tracking motion;
2. Wall contact motion;
Perspectives:

Nonlinear systems e.g. master:

\[ M(\theta)\ddot{\theta}(t) + C(\theta, \dot{\theta})\dot{\theta}(t) + g(\theta) = F_h(t) + F_m(t). \] (86)

One master/multiple slaves teleoperation system;
Perspectives:

Delay-scheduled state-feedback controller design:

\[
K(\tau(t)) = K_0 + K_1 \tau(t) + K_2 \tau^2(t) + \cdots + K_h \tau^h(t) = \sum_{i=0}^{h} K_i \tau^i(t).
\]

Switch control strategy between the passivity-based approach and the approach proposed in this thesis.
Perspectives:

- Global stability analysis with perturbation observers;
- Dynamics of the human operator and the environment;
Thank you for your attention! 😊